STREAM TUBE MODEL FOR WATER QUALITY PREDICTION IN MIXING ZONES OF SHALLOW RIVERS

WATER RESOURCES PAPER NO.14

STREAM TUBE MODEL FOR WATER QUALITY PREDICTION IN MIXING ZONES OF SHALLOW RIVERS

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July 1980

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Additional copies of this Water Resources Report" may be obtained in person from the Ontario Government Bookstore, 880 Bay Street, Toronto, Ontario or by mail from the Ontario Government Publications Centre, 5th Floor, 880 Bay Street, Toronto, M5S 1Z8.

ISSN 0708-4749
ISBN 0-7743-6844-6

Gowda, T.P.H.,1980. Stream Tube Model for Water Quality Prediction in Mixing Zones of Shallow Rivers, Water Resources Paper No. 14, Ontario Ministry of the Environment, Water Resources Branch.

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ABSTRACT

Wastewater effluents discharged into streams and rivers contain pollutants such as residual chlorine, ammonia and phenol which have the potential to be toxic to aquatic biota. Generally, regulations adopted by water management agencies provide for mixing zones in the vicinity of outfalls to accommodate the conflicting needs of effluent discharge and protection of the biota. The mixing zone consists of a limited use zone (LUZ) in a portion of the cross-section of the river where pollutant concentrations may exceed allowable levels, and a zone of passage (ZOP) in the remaining portion which serves as a suitable habitat for the biota. This report deals with the development and application of mathematical models, based on the stream tube concept, to predict the spatial distributions of pollutants of concern in mixing zones of shallow rivers below bank outfalls.

The stream tube model formulations consider partial cumulative discharge instead of the lateral distance as the transverse co-ordinate. Analytical solutions of the steady state, 2-D convection-diffusion equation given in the literature, are modified to account for the longitudinal variabilities of decay and dispersion parameters. The formulations are applicable to a natural stream stretch in which the discharge remains constant.

The concentration of a pollutant along a given lateral boundary of LUZ reaches a maximum at some longitudinal distance below the outfall; this point is termed "critical point". Analytical expressions have been derived to determine the co-ordinates of the critical point, as well as the allowable effluent concentration to meet a specified instream concentration criterion. A family of curves is presented for a range of dimensionless co-ordinates of the critical point.

Two computer programs, developed for application to streams and rivers in which the instream process parameters are considered to be reach-dependent, have been documented. One of the programs,

MIXCALBN, is suitable for model calibration and verification studies, whereas the other, MIXAPPLN, is suitable for predicting the effects of various management options on the allowable effluent concentration and maximum longitudinal boundary of LUZ.

A general procedure for carrying out field studies is described. The computational procedures involved in the analysis of field data are outlined. Dimensionless expressions are developed to test whether the dispersion characteristics in a river are a function of width or depth, and to obtain dimensionless coefficients from graphical plots. A computer program MIXANDAT, developed to aid in the analysis of the field data, is described and documented in this report.

The field data collected during two surveys in the Grand River below the Waterloo water pollution control plant (WPCP) effluent outfall are utilized to study the dispersion characteristics of the river there. During these surveys, data were obtained on the cross-sectional distributions of rhodamine WT dye (injected continuously into the effluent at the outfall), chloride ion and dissolved solids (measured by specific conductance). The dye distributions showed a gradual shift in the position of peak concentration away from the discharge bank; whereas the chloride ion and conductance peaks occurred at or near the discharge shoreline. However, the effluent plume widths at various transects and nondimensional dispersion coefficients, determined from the individual tracer distributions, were found to be in general agreement with one another. The effluent plume width at a transect was also found to be a function of the channel width at that transect. Results of data analysis indicate that the transverse dispersion coefficient is a function of channel width rather than the depth in the study stretch of the river.

The chloride ion distribution data collected in the Grand River during the two surveys were used to calibrate and verify the stream tube model satisfactorily. The model was then used to predict the effect of typical management options on allowable effluent

concentration of total ammonia and associated maximum longitudinal spread. The results indicate that, in general, the allowable effluent concentration as well as the maximum longitudinal spread of a LUZ would increase due to: (a) an increase in the lateral boundary of the LUZ; (b) an increase in the streamflow rate; and (c) a decrease in temperature.

Data on the distribution of residual chlorine in the Boyne River below Alliston were used to validate the model satisfactorily for a nonconservative material. For comparison purposes, allowable effluent concentrations of residual chlorine to meet a specified instream criterion were predicted by the LUZ concept and on the assumption of instantaneous complete mixing (ICM) of effluent with the streamwater. In general, the results indicate that in some cases, the ICM assumption or the dilution ratio concept may lead to a LUZ with too large a lateral boundary (and consequently, too small a zone of passage) due to an underestimation of treatment required in comparison to the predictions based on the LUZ concept; whereas, in some other situations, the predictions based on the former method may result in treatment requirements that are too stringent in comparison to those of the latter method. Thus, in the case of a mixing zone below a bank outfall, in which lateral mixing of effluent is gradual, it is inappropriate to determine the allowable effluent concentration of a pollutant based on the ICM assumption.

ACKNOWLEDGEMENTS

The author expresses his deepest gratitude to the following persons (listed in alphabetical order) for a voluntary review of the manuscript, and for providing constructive criticisms and suggestions:

Dr. S. Beltaos, Research Scientist, National Water Research Institute, Canada Centre for Inland Waters, Burlington, Ontario, Canada.

Professor D.R.F. Harleman, Ford Professor of Engineering and Director, Ralph M. Parsons Laboratory for Water Resources and Hydrodynamics, Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts, U.S.A.

Dr. B.G. Krishnappan, Research Scientist, National Water Research Institute, Canada Centre for Inland Waters, Burlington, Ontario, Canada.

Dr. Y.L. Lau, Head, Environmental Hydraulics Section, National Water Research Institute, Canada Centre for Inland Waters, Burlington, Ontario, Canada.

Professor R.G. Warnock, Department and Civil Engineering, University of Ottawa, Ottawa, Ontario, Canada.

Dr. N. Yotsukura, Research Hydrologist, United States Geological Survey, Department of the Interior, Reston, Virginia, U.S.A.

The manuscript was also reviewed by the Chairman and members of Working Group II, Water Management in Ontario-Implementation, and several staff members of the Ontario Ministry of the Environment. Special thanks are due to Messrs. D. N. Jeffs, F. C. Fleischer, D. G. Weatherbe and Dr. I.W. Heathcote for their helpful suggestions during the course of this investigation.

The assistance of Messrs. L. Van Biesbrouck and D. Alleway, formerly with the River Systems Unit, Water Modelling Section, in carrying out field studies is gratefully acknowledged. Thanks are also due to Ms. L. Post and Mr. A. Bacchus for providing computational and drafting assistance. The manuscript was typed by the Word Processing Centre of the Water Resources Branch.

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The following notations are used in this report:

A:	Cross-sectional area of stream channel.
A_c:	Area under the c-y curve.
A_q:	Area under the c-q curve.
B:	Weighted mean value of width of stream channel.
b:	Top width of water surface at a given transect.
bex:	Width exponent in Leopold-Maddock Equation.
c:	Concentration of a material at a given point in mixing zone.
c_a:	Completely mixed instream concentration of a material.
c_b:	Background concentration in river water.
c_c:	Concentration of a conservative material at a given point.
c_e:	Effluent concentration of a material.
c_k:	Depth-averaged concentration at a lateral distance, y_k, from outfall shoreline.
c_L:	Critical concentration.
c_R:	Ratio (c_c/c_a).
c_r:	Ratio (c_L/c_a).
c_s:	Specified instream concentration criterion.
c_t:	Concentration of total ammonia.
c_u:	Concentration of un-ionized ammonia
c_{eA}:	Allowable effluent concentration
c_{mz}:	Instream concentration at the boundary of theoretical mixing zone.
c_{ra}:	Ratio (c/c_{atr})
c_{wa}:	Instream concentration at a downstream boundary due to effluent discharge.
c_{atr}:	Average concentration of material at a given transect obtained from (mass flux/discharge).
c_{max}:	Maximum concentration of material at a given transect.
c_{crit}:	Critical concentration estimated during iteration procedure.
D_y:	Diffusion factor.
E_L:	Nondimensional term defined in Eq. 27.
e_y:	Transverse dispersion coefficient.
F:	Net mass flux at a given transect.
f_k:	Mass flux of material in a vertical strip, k.

f_x: Relative change in x, (dx/x), given by Eq. 32.

f_1: Constant in the resistance equation (Eq. 46b).

f_2: Exponent in Eq. 46b.

G_L: Nondimensional term defined in Eq. 27.

h: Local depth of water at a lateral distance, y.

$\bar{H}$: Mean depth in a given stretch of river.

$\bar{h}$: Mean depth at a given transect.

h_k: Local depth at a distance, y_k, from outfall shoreline.

hex: Depth exponent in Leopold-Maddock Equation.

k_i: Decay rate in i^{th} reach of river.

ka: Dissociation constant for ammonia.

k_b: Decay rate associated with the background concentration of nonconservative material.

k_d: Decay rate of nonconservative material in river.

k_p: Attenuation rate of a material due to transverse dispersion.

k_t: Gross decay rate (due to dispersion and chemical reaction).

k_{av}: Weighted mean value of the decay rate, k_d.

L*: Characteristic length.

m_x: Metric coefficient (or scaling factor) associated with x-axis.

$\bar{m}_x$: Average value of m_x at a given transect.

m_y: Metric coefficient associated with y-axis.

n: Number of images.

n': Manning's roughness coefficient.

p: Ratio (q/Q).

p_L: Ratio (q_L/Q).

p_m: Maximum value of p at which n is zero.

p_s: Ratio (q_s/Q).

pH: pH value of water.

pka: log (1/ka).

Q: Discharge in river below outfall.

Q_e: Effluent flow rate.

q: Cumulative partial discharge measured from a reference bank of stream channel.

q_L: Lateral boundary of limited use zone.

q_s: Cumulative partial discharge between reference bank and effluent discharge point in river.

R_i: Decay factor in i^{th} reach of stream.

R': Decay factor given by Eq. 4.

R": Factor derived from a chemical equilibrium relationship.

r: Hydraulic radius.

s_e: Energy gradient.

T^o: Temperature of stream water.

t: Time of travel.

U: Weighted mean velocity of streamflow.

$\bar{U}$: Mean velocity of flow in a stretch of river.

U^*: Characteristic velocity.

u: Depth-averaged local velocity at a lateral distance, y (Eq. 3).

u_k: Depth-averaged local velocity at a distance, y_k, from outfall shoreline.

u_*: Bed shear velocity.

$\bar{u}$: Mean velocity at a given transect.

uex: Velocity exponent in the Leopold-Maddock equation.

x: Longitudinal distance below outfall.

x_L: Critical distance.

x_w: Longitudinal distance from outfall to a downstream boundary.

x_{mz}: Theoretical mixing zone length.

x_{sce}: Longitudinal distance along the discharge shoreline where the concentration is c_s, as a result of a given effluent concentration, c_e.

x_{crit}: Critical distance estimated during iteration procedure.

x_{sceA}: Value of x_{sce} when effluent concentration is c_{eA}.

y: Lateral distance measured from outfall shoreline.

α_1: Dimensionless coefficient in Elder's equation, (e_y/ru_*).

α_2: Dimensionless dispersion coefficient, $(e_y/h\bar{u})$.

α^*: Nondimensional coefficient, (e_y/L^*U^*).

β: Dimensionless coefficient given by Eqs. 14a-c.

β_e: Dimensionless dispersion coefficient, $(e_y/b\bar{u})$.

δ: Numerical value of n^{th} term of exponential series.

ϕ: Dimensionless term (Eqs. 6 & 13).

ϕ_L: Ratio (G_L/E_L).

ψ : Shape-velocity factor.

σ_y^2: Variance of c(x,y) versus y distribution curve.

σ_f^2: Variance of unit flux distribution curve.

σ_q^2: Variance of c(x,q) versus q distribution curve.

θ : Temperature correction factor.

I. INTRODUCTION

1.1 General

Wastewater effluents discharged into streams and rivers contain pollutants such as residual chlorine, ammonia and phenol which have the potential to be toxic to aquatic biota. The toxic effects of such pollutants are minimized by suitable siting of the outfalls in receiving water bodies. Generally, pipe outfalls located at a bank or in a cross section at some known distance away from the bank are employed in shallow rivers. In such cases, the dilution of effluent with streamwater is gradually increased in the longitudinal direction, resulting in proportional reduction of the effluent concentrations of pollutants. At the same time, concentration gradients are set up in the longitudinal, lateral and vertical directions. Ultimately, the cross sectional concentration distribution attains uniformity at some distance below the outfall. The zone of the receiving stream between the outfall and the nearest cross section at which the concentration distribution becomes uniform is generally known as "mixing zone", and the corresponding distance is termed "mixing zone length". The distance below the outfall at which the effluent spreads across the entire width of the river is termed "crossing distance" (13)[1]. Within the mixing zone, a portion of the cross section of the river in which the concentration of a pollutant of concern exceeds a specified instream criterion is termed the "limited use zone" (LUZ); the remaining portion, which is designated to serve as suitable habitat for fish and other desirable aquatic life is known as the "zone of passage" (ZOP) (15, 19, 24).

The salient features of a mixing zone are depicted in Fig. 1. Usually, the location of the lateral boundary is designated as a fraction of channel width or river discharge for simplicity, which indicates that the shape of LUZ would be a rectangle. However, the concentration decreases in the downstream direction due to dispersion and decay (see Fig. 3 in Chapter II), and hence, the lateral boundary of LUZ will be located along a specified criterion concentration contour line as indicated by the broken line in Fig. 1.

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1 Numbers in parentheses indicate serial numbers of references.

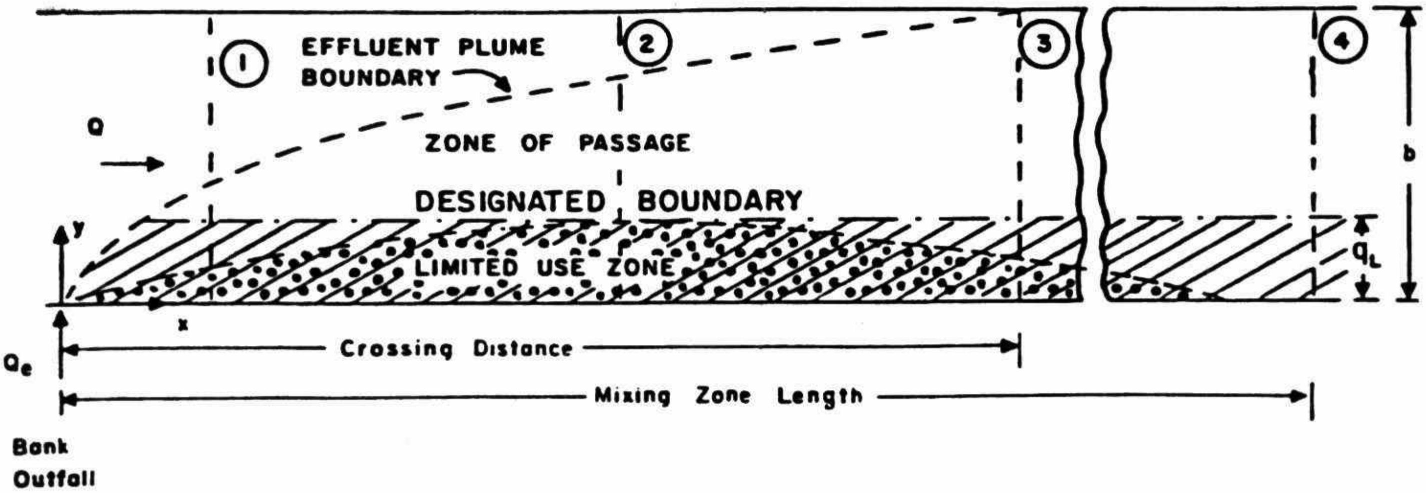


FIGURE 1. SCHEMATIC VIEW OF MIXING ZONE

Several water management agencies have adopted guidelines and criteria for water quality in mixing zones of rivers (for example, see References 19 and 24). In the application of such guidelines and criteria, water resource managers would like to know answers to the following questions:

1. Does a given combination of effluent discharge, Q_e, effluent concentration, c_e and river discharge, Q, result in compliance with a specified criterion, c_s, for a given lateral boundary and what is the maximum longitudinal spread, x_{sce}, at which c_s is met?

2. If the conditions stated above result in noncompliance with the criterion, then what is the allowable effluent concentration, c_{eA} and the corresponding x_{sceA}?

3. What are the effects on c_{eA} and x_{sceA} of seasonal variations in Q_e, Q and temperature, as well as pH in the case of ammonia?

To provide answers to the above questions, transverse dispersion models coupled with pertinent water quality process kinetic expressions can be useful tools.

Paily and Sayre (20) and Stefan and Gulliver (23) have described the development and application of transverse dispersion models to mixing zones in rivers associated with thermal and radioactive waste effluents. The application of dispersion models coupled with process kinetics to predict the spatial distributions of dissolved oxygen, total residual chlorine (TRC) and toxic (un-ionized) ammonia in mixing zones of shallow rivers has been presented by Rood and Holley (21) and the writer (8, 10, 11). These models were formulated for rectangluar channels thus limiting their usefulness for the nonprismatic channels commonly encountered in natural streams and rivers. However, the development of models based on the "stream tube" approach by Yotsukura and Cobb (29) has overcome the major limitations of the models that were applicable to rectangular and other prismatic channels.

The formulations and procedures presented in this report are mainly applicable to the prediction of concentration distributions in the far-field regions of mixing zones below bank outfalls, although some of the procedures are generally valid for pipe outfalls located anywhere in the cross-section of a river. The case studies presented in this report deal with the modelling of a conservative material (chloride ion), as well as two nonconservative pollutants (total ammonia and TRC). However, the models are also applicable to phenol, radionuclides, indicator bacteria (e.g. coliform group) and other pollutants whose transport characteristics in rivers are governed by advection, transverse dispersion and first order decay.

1.2 Objectives

The objectives of the study are as follows: (i) to modify the stream tube model equations presented by Yotsukura and Cobb to account for longitudinal variabilities in decay rate coefficients and hydraulic parameters of a channel; (ii) to develop analytical expressions and procedures for computing critical point co-ordinates, allowable effluent concentration and maximum longitudinal spread of LUZ; (iii) to calibrate and verify the modified model using field data on the distribution of a conservative substance gathered in a shallow, wide natural river; (iv) to validate the applicability of the model to the prediction of TRC in a shallow stream; and (v) to predict allowable effluent concentrations of total ammonia and maximum longitudinal spread under various management options.

II. MODEL DEVELOPMENT ASPECTS

2.1 Basic Stream Tube Model

The fundamental concept of the stream tube model developed by Yotsukura and Cobb (29) lies in considering the cumulative partial discharge, q, at a given cross-section instead of the lateral distance, y, as independent variable. In this approach, the cross section is divided into a number of vertical strips termed "stream tubes", such that the discharge within each stream tube is the same; whereas, in the traditional approach, the strips are of equal width. Thus, the cross sectional concentration distributions, c (x, q), predicted by the stream tube model will be functions of q. These distributions can be transformed into the traditional c(x, y) versus y plots by knowing the relation between q and y at each transect.

The derivations of the basic equations of the stream tube model have been presented by Yotsukura and his co-workers (28, 29, 30, 31); they are subject to the following assumptions:

1. The density of effluent (or solute) is the same as that of the receiving water. This assumption is reasonably satisfactory for most of the municipal effluent discharges to surface water bodies.

2. The concentration distributions in the far-field are not affected by the near-field mixing processes (eg. dilution due to initial momentum of jet). Usually, the jet-induced diffusion approaches the ambient diffusion in a short distance below a source in a shallow river.

3. The vertical distribution of effluent (or solute) in the river channel is uniform. Generally, the longitudinal distance required to attain vertical uniformity is fairly short in shallow rivers, being on the order of 50 to 100 times the channel depth; thus, the assumption is justified.

4. The transport due to longitudinal dispersion is negligible. In the case of continuous effluent discharge, this transport is very small in relation to that due to convection and lateral dispersion, thus justifying the assumption.

The simplified two-dimensional convection-diffusion equation, describing the distribution of a nonconservative material in the far-field region of the mixing zone in a shallow river under steady state conditions, can be written in the following form (assuming that the decay of the material follows first-order kinetics):

$$\frac{\partial c}{\partial x} = D_y \frac{\partial^2 c}{\partial q^2} - \frac{m_x k_d c}{u}, \tag{1}$$

in which x = longitudinal distance below the source; q = cumulative partial discharge measured from a reference bank; c = concentration of pollutant at a point (x, q); u = depth-averaged local velocity of flow in the x-direction; k_d = first-order decay rate coefficient (assumed to be independent of x and y); D_y = diffusion factor; and m_x is a metric coefficient (or scaling factor) introduced to correct for differences between longitudinal distances along curved co-ordinate surfaces and those measured along the x-axis, which occur due to curvature in channel alignment and/or variations in width (30). The diffusion factor is assumed to be constant at a cross-section and is given by

$$D_y = \overline{(m_x e_y u h^2)} \quad ; = \frac{1}{Q} \int_0^Q (m_x e_y u h^2)\, dq , \tag{2}$$

in which the overbar indicates the average value of the product term; h represents the local depth of water at a lateral distance, y, (the latter being measured from the reference bank); e_y = transverse dispersion coefficient; and Q = discharge in the river just below the source. The diffusion factor can also be expressed in terms of the bulk flow parameters of channel as outlined

elsewhere (see Section 2.2). The independent variable, q, is defined by

$$q = \int_{o}^{y} m_y \; uh \; dy, \qquad (3)$$

in which m_y is a metric coefficient associated with y-axis. From Eq. 3, it is readily seen that q = o at y = o and q = Q at y = b, where b denotes the channel width at a given cross-section of the river. The values of m_x and m_y might vary from one point to another, except that $m_x = 1$ on the x-axis and $m_y = 1$ on the y-axis. Methods of determining the values of the metric coefficients are presented by Yotsukura and Sayre (30). For the sake of simplicity m_x and m_y are assumed to have unit values in further formulations (with some exceptions where they are retained for clarity).

Yotsukura and Cobb (29) have presented closed-form analytical solutions of Eq. 1 applicable to far-field distributions of a conservative material (i.e., k_d=0), discharged as continuous point and horizontal line sources. The concentration of a nonconservative material at a given point in the far-field is obtained by multiplying the right-hand side of the analytical expressions by a factor, R', as outlined by Dobbins (3), where

$$R' = \exp\,(-\,k_d x/\bar{U}) \qquad (4)$$

in which $\bar{U}$ = average velocity of flow in a river stretch, on the assumption that the term $(m_x k_d/u)$ appearing in Eq. 1 can be approximated by $(k_d/\bar{U})$. The latter approximation is suggested to be satisfactory for most practical situations (personal communications, N. Yotsukura; and Y.L. Lau). The solution of Eq. 1 for a nonconservative pollutant discharged from a pipe outfall (i.e., point source) located at x=o and q=q_s is then given by

$$c(\phi, p) = R' \ c_a \quad (4 \ \pi \ \phi)^{-\frac{1}{2}} \left\{ \sum_{n=o}^{+\infty} \left[\exp\left\{- \frac{(2n+p_s-p)^2}{4\phi}\right\} \right. \right.$$

$$\left. + \exp\left\{- \frac{(2n+p_s+p)^2}{4\phi}\right\}\right] + \left. \sum_{n=1}^{+\infty} \left[\exp\left\{- \frac{(2n-p_s-p)^2}{4\phi}\right\} + \exp\left\{- \frac{(2n-p_s+p)^2}{4\phi}\right\}\right]\right\} \quad (5)$$

in which n=number of images required to account for the effect on concentration of the reflection of the material from the channel banks; and ϕ , p, p_s, c_a are defined by:

$$\phi = D_y x/Q^2; \quad p = q/Q; \quad p_s = q_s/Q; \quad c_a = c_e \ Q_e/Q, \qquad (6)$$

where c_e and Q_e represent the effluent concentration of the pollutant and effluent flow rate, respectively. Note that c_a denotes the cross-sectional average concentration in the river just below the outfall. (An alternative solution of Eq. 1 for the case of bank outfall is given in Section 2.3).

Eq. 5 is applicable to a point source located anywhere in the cross section of a river as described above; for the particular case of discharge from a bank outfall, $p_s = 0$. The analytical solution of Eq. 1, given by Eq. 5, is subject to the conditions $c_e \rightarrow \infty$ and $Q_e \rightarrow 0$; thus, Eq. 5 is generally valid when Q_e is negligibly small relative to Q. When these conditions are not satisfied, the values of c(x,q) predicted by Eq. 5 may be higher than c_e, although such situations do not occur in real world. An empirical method outlined in subsection 2.3.2 can be used to examine whether the predicted concentration at a given point exceeds c_e, on the assumption that the pollutant is conservative. Such anomalous predictions, generally confined to small values of x, should be ignored in the analysis.

It should be noted that the concentrations of some pollutants (e.g. un-ionized ammonia) are also dependent on the chemical equilibrium

of solutions. In such cases, the pollutant concentrations are obtained by multiplying the predictions of Eq. 5 by yet another factor, R", derived from chemical equilibrium considerations. For example, in the case of ammonia, the predictions of Eq. 5 would represent the concentrations of total ammonia (i.e. the sum of ionized and un-ionized forms of ammonia). The concentration of un-ionized ammonia is a function of pH, temperature, $T^{o}C$, and concentration of total ammonia, c_t. The relation between c_u and c_t can be written in the form $c_u = R''c_t$, where the factor R" is given by (10, 11, 26):

$$R'' = 1.0/(1 + 10^{pka - pH}), \tag{7}$$

where $\quad pka = 0.09018 + (2729.92/(T^{o} + 273.2)),$

in which ka = dissociation constant for ammonia, and pka = -log ka. This development is based on the assumption that the instream temperature and pH distributions at various cross-sections are uniform. This assumption is reasonably satisfactory for most practical situations.

2.2 Reach-Dependancy of Parameters

In water quality investigations, it is a common practice to divide a study stretch of the river into a number of longitudinal segments, each segment being termed a "reach" such that the most dominant parameters affecting water quality in each reach are fairly constant, but differ from one reach to another. The important parameters having longitudinal variabilities of concern in mixing zone studies include the width, depth, velocity, decay rate coefficient and nature of stream bed material. Such variabilities can be incorporated into Eq. 5 through the development of modified expressions for decay function and dispersion factor.

In order to derive the modified expressions, it is assumed that the concentration profiles at the boundaries of successive transects are continuous. If field measurements indicate that the concentration

profiles are subject to abrupt changes (e.g. a shift in the position of peak concentration at successive transects), then this model is not likely to be applicable; in such cases, numerical solutions of Eq. 1 may be necessary.

The development of modified expressions for the decay function and dispersion factor is outlined below.

2.2.1 Decay Function:

Let u_i and k_i represent, in order, the mean velocity and decay rate in the ith reach of length $(x_i - x_{i-1})$, where x_{i-1} and x_i denote the longitudinal distances from the source to the upstream and downstream edges of the reach. The time of travel, t_i, in the ith reach is given by $(x_i - x_{i-1})/u_i$. The decay function in the reach, R_i, is dependent on k_i and u_i, both of which are functions of x only. Thus, if C_o and C_1 are the concentrations at the upstream and downstream edges of the first reach, then $C_1 = R_1 C_o$ where $R_1 = \exp(-k_1 t_1)$; the concentration, C_2, at the downstream edge of the second reach is given by $C_2 = R_2 C_1 = R_1 R_2 C_o$, where $R_2 = \exp(-k_2 t_2)$. A general expression for the concentration, C_i, at the downstream edge of the ith reach can then be written in the compact form

$$C_i = C_o \left(\prod_{j=1}^{i} R_j \right), \tag{8}$$

in which $R_i = \exp(-k_i t_i)$; and $\left(\prod_{j=1}^{i} R_j \right) = R_1 R_2 \ldots R_i$, denoting the product of the R-terms. Thus, when the decay rate is reach-dependent, R' in Eq. 5 must be replaced by the product term.

Some of the computations described later (viz., iterative procedure for critical co-ordinates and longitudinal spread) require the use of a moving average value of the decay rate, k_{av}. The average value between the outfall and m^{th} transect is determined from

$$k_{av} = -\frac{1}{T_m} \log_e\left(\prod_{j=1}^{m} R_j\right) \tag{9}$$

in which $T_m = (t_1 + t_2 \ldots\ldots + t_m)$ denotes the total travel time from the outfall to the transect, m. Alternatively, k_{av} can also be calculated from $\Sigma(k_m t_m)/T_m$.

2.2.2 Diffusion Factor:

In the formulation of the basic stream tube model by Yotsukura and Cobb (29), the diffusion factor, D_y, at any transect is assumed to be constant, but could vary from one transect to another. The values of D_y at various transects in natural streams were obtained by a model validation procedure using Eq. 5 in conjunction with field data. Then, the values of the transverse dispersion coefficient, e_y, at each transect were evaluated from Eq. 2 by computing the values of $\overline{uh^2}$ from measured depth and velocity profiles (29). The values of D_y can also be estimated from the variances of c(x,q) versus q distributions, as outlined by Beltaos (1) and Yotsukura and Cobb (29); the latter is dealt with in Chapter III. The effect of variation of channel hydraulic parameters on D_y, and development of some simplified expressions are presented below.

Let us consider the first reach immediately below the source. Following Beltaos (1,2), an expression for D_y can be written in the form

$$D_y = \psi \; e_y \, \bar{u} \, \bar{h}^2 \quad , \tag{10}$$

in which $\bar{h}$ and $\bar{u}$ denote the cross-sectional mean values of depth and velocity, respectively; and ψ denotes a "shape-velocity" factor (introduced to account for the deviations of local depths and velocities from the cross-sectional mean values).

The shape-velocity factor can be evaluated from the relationships presented by Beltaos (2). An expression which relates ψ to cross-sectional depth and velocity profiles is presented in Section 3.2. The results presented by Beltaos indicate that the value of the factor for a rectangular channel is 1.0, whereas for natural streams, the value is likely to be in the range 1.0 to 3.2. Yotsukura and Cobb (29) also took account of the nonuniformity factor by considering the nondimensional ratio $(\bar{u}\bar{h}^2/\overline{uh^2})$, which is the inverse of ψ. The values of the ratio were in the range 0.35-0.83; in terms of ψ, the range is 1.2 to 2.9, which is in good agreement with the range given by Beltaos. A comparison of Eqs. 2 and 10 indicates that an average value of the metric coefficient, $\bar{m}_x$, needs to be included in Eq. 10; however, none of the above results include the influence of the metric coefficient.

By substituting $Q = b\bar{u}\bar{h}$ and $D_y = \psi\, \bar{m}_x\, e_y\, \bar{u}\bar{h}^2$ in the expression for ϕ in Eq. 6, the following relation is obtained:

$$\phi = \psi\, \bar{m}_x\, e_y x/b^2\, \bar{u} \tag{11}$$

A review of the literature indicates that the transverse dispersion coefficient, e_y can be related to the bulk flow parameter of the channel in several ways (6, 9, 13, 14, 15); some of those relationships are presented in Chapter III (see subsection 3.3.1). In general, the relationships are of the form $e_y = \alpha^* L^* U^*$, where L^* denotes a characteristic length (viz., width, depth or hydraulic radius of channel), U^* denotes a characteristic velocity (viz., mean velocity of flow or bed shear velocity) and α^* is a nondimensional dispersion coefficient. By substituting for e_y in Eq. 11, we get

$$\phi = \left[\bar{m}_x \quad \psi \quad \alpha^* \quad \frac{L^*}{b} \quad \frac{U^*}{\bar{u}}\right]\left(\frac{x}{b}\right) \tag{12}$$

The factor appearing inside the square brackets in Eq. 12 is dimensionless, consisting of the geometric and hydraulic parameters of the first reach below the source. If we denote this factor by a

nondimensional coefficient, β , then Eq. 12 can be written in the following form:

$$\phi = \beta\ x/b \tag{13}$$

The expression for ϕ given by Eq. 13 is much simpler than that appearing in Eq. 6. Eq. 13 should be preferred for use in modelling studies since it circumvents the need to compute D_y and e_y separately. By equating the expressions for ϕ appearing in Eqs. 6 and 13, we get

$$\beta = (D_y b/Q^2) \ ; \ = (D_y/b\bar{u}^2\ \bar{h}^2) \tag{14a}$$

A comparison of the expressions for ϕ appearing in Eqs. 11, 12 and 13 results in

$$\beta = (\bar{m}_x\ \psi\ e_y/b\bar{u}) \ ; \ = (\bar{m}_x\ \ \psi\ \alpha^* L^* U^* /b\bar{u}) \tag{14b}$$

The foregoing expressions indicate that β is a function of the basic channel geometric and hydraulic parameters. If we assume that the transverse dispersion coefficient, $e_y = \beta_e bu$, following Lau and Krishnappan (16), we obtain the following simplified expression for β from Eq. 14b:

$$\beta = \bar{m}_x\ \psi\ \ \beta_e \ , \tag{14c}$$

in which β_e is a dimensionless dispersion coefficient. (Note: The various relations of e_y to bulk flow parameters of channel are considered further in Chapters III and IV). In Eq. 14c, $\bar{m}_x$ and ψ may be dependent on the streamflow rate, whereas β_e is likely to be a function of the channel bed roughness, turbulence, velocity distribution, etc. Thus, the value of β in a given channel reach may be a function of some of those parameters; however, the nature of relationship is not known. For practical purposes, it will be assumed that the value of β for a given reach remains constant under all streamflow conditions. Methods of estimating β_e and β from field data are outlined in Chapter III.

The diffusion factor, D_y, introduced by Yotsukura and Cobb(29), and the dimensionless coefficient, β , defined in Eqs. 14a-c, are both indicative of the transverse dispersion characteristics in the context of the stream tube model formulations. Thus, it appears appropriate to use the term "nondimensional diffusion factor" for β in anolgoy with the term "diffusion factor" for D_y. However, the theoretical basis of the expressions for β may need further examination (personal commuinication, N. Yotsukura).

The various expressions for ϕ , D_y and β presented above are applicable to the first reach of the stream channel immediately below the source. When the geometric and hydraulic parameters appearing in those expressions vary in a given stream stretch, then the moving average values of the pertinent parameters should be used in the computations. An examination of the various equations indicates that the value of ϕ required to solve Eq. 5 can be computed from Eq. 13; and β and b appearing in the latter equation are likely to be dependent on x. Thus, the following modified form of Eq. 13 should be utilized:

$$\phi_i = \beta_i x_i / B_i \tag{15}$$

in which B_i denotes the moving average value of channel width from the source to a transect, i, located at a distance x_i (see Eq. 23); and β_i represents the average value of the dimensionless coefficient applicable to the channel stretch of length, x_i.

2.3 Modified Stream Tube Models

2.3.1 General Solution for Pipe Outfalls:

By incorporating the foregoing modified expressions for decay function and dispersion factor in Eq. 5, we obtain the following equation applicable to a pipe outfall located at any point in a cross-section of river:

$$c(\phi_i, p) = \left(\prod_{j=1}^{i} R_j\right) c_a \quad (4 \pi \phi_i)^{-\frac{1}{2}} \left\{ \sum_{n=o}^{\infty} \left[\exp\left\{- \frac{(2n+p_s-p)^2}{4 \phi_i}\right\} + \exp\left\{- \frac{(2n+p_s+p)^2}{4 \phi_i}\right\}\right] + \sum_{n=1}^{\infty} \left[\exp\left\{- \frac{(2n-p_s-p)^2}{4 \phi_i}\right\} + \exp\left\{- \frac{(2n-p_s+p)^2}{4 \phi_i}\right\}\right]\right\} \quad (16)$$

In this equation, all parameters are known with the exception of the number of images, n. A simple method of estimating n has been outlined previously by the writer (10, 11). In this method, the n^{th} exponential term of each series of Eq. 16 is set to a number, δ , such that the value of the nth term is small enough to cause negligible effect on $c(x_i, q)$ for given values of p, p_s and ϕ_i; the resulting algebraic equation is solved for n. For example, if the nth term of the first series of Eq. 16 is set to δ , then the following relationships are obtained:

$$\exp\{ - (2n+p_s-p)^2/4 \phi_i \} = \delta \quad (17)$$

$$n = 0.5(p-p_s) \pm \sqrt{\phi_i \log_e(1/\delta)} \quad (18)$$

From Eq. 18, two values of n are obtained: A lower bound, n_1, generally less than or equal to zero with the negative sign, and a higher bound, n_2, greater than or equal to zero with the positive sign. The total number of images, n_t, is given by the sum of the absolute values of n_1 and n_2. The summation of the first series is then carried out from n=0 to $n=n_t$. Similar procedures can be followed to solve the other three series of Eq. 16. A trial and error procedure indicated that the total number of images estimated with $\delta = 8.5 \times 10^{-4}$ was sufficient to determine the truncation point of each series for all values of ϕ_i, p and p_s normally encountered in practice. The predictions of Eq. 16 for the case of an example cited by Verboom (25) were in good agreement with the results reported by that author (see Appendix A).

A computer program MIXCALBN has been written in FORTRAN language to predict concentration distributions using Eq. 16, the details of which are described in Section 2.10. (Note: As stated earlier, some values of c(x,q) predicted by Eq. 16 may be higher than c_e; an empirical method for examining the occurrence of such conditions is outlined in the following subsection.)

2.3.2 Simplified Solution for Bank Outfalls:

As previously stated, p_s = 0 in Eq. 16 for the particular case of discharge from a bank outfall. A particular analytical solution of Eq. 1 applicable to the bank outfall case can be obtained by following Rood and Holley (21), the latter solution being much simpler than Eq. 16; the equation can be written in dimensionless terms as follows:

$$c_R\ (\phi_i,\ p) = \left(\prod_{j=1}^{i} R_j\right)(\ \pi\ \phi_i)^{-\frac{1}{2}} \sum_{n=-\infty}^{\infty} \exp\{-(p-2n)^2/4\ \phi_i\}, \qquad (19)$$

in which $c_R(\phi_i,\ p) = c(\phi_i,\ p)/c_a$. The number of images, n, can be determined as outlined above from Eq. 18 with p_s=0; the summation of series in Eq. 19 is carried out from n = $-n_1$ to $+n_2$. It should be noted that the predictions of Eq. 19 will be identical to those of Eq. 16 with p_s = 0 for a given set of parameters. For the example cited by Verboom, the predictions of Eq. 19 were comparable to the results tabulated by that author (25).

An important use of Eq. 19 lies in its suitability to derive simplified expressions for the critical point of a LUZ by following the procedures described by the writer previously (11); the derivation is presented in Section 2.5. Eq. 19 is also useful to derive an expression required in calculating the maximum longitudinal distance below the outfall at which the specified criterion for a pollutant is met; this will be outlined later in Section 2.7.

For a conservative material, the decay function becomes unity in Eq. 19 (since $k_d=0$ in each reach); then it is possible to calculate the values of c_R for given values of ϕ and p. A graphical solution of Eq. 19, obtained for a range of values of ϕ_i normally encountered in practice with p=0 to 1.0, is presented in Fig. 2. An examination of this dimensionless plot shows that $c_R = 1.0$ at $\phi = 1.0$ for all values of p. (Note: In Fig. 2, c_R values appear to be 1.0 for $\phi \leqslant 0.6$, because of the close proximity of the data points.) This indicates that the cross-sectional concentration distribution of a conservative material attains uniformity at $\phi = 1.0$, thus defining the theoretical mixing zone length for a conservative material. Therefore, when $\phi < 1.0$, two-dimensional analysis is required whereas for $\phi \geqslant 1.0$, one-dimensional analysis will suffice, based on theoretical considerations.

The graph presented in Fig. 2 can be used to obtain the cross-sectional concentration distribution of a conservative material at any transect in the mixing zone. The corresponding concentration of a nonconservative material subjected to first order decay can then be obtained by multiplying each concentration value by the pertinent decay factor. When the velocities and decay rates are reach-dependent, the computations are laborious and time-consuming. However, through the use of weighted mean values k_{av}, B and U for the stretch as an approximation (Eqs. 9, 23 and 24), and by estimating β as outlined in Section 3.3, the following procedure can be used to obtain concentration distributions from Fig. 2:

1. Select the design case parameters, viz., Q, Q_e, c_e and temperature.

2. Estimate β, k_{av}, B and U.

3. Compute ϕ for known x, and determine c_a and p using appropriate relationships given in Eq. 6.

4. From Fig. 2, read values of c_R for various values of p corresponding to ϕ computed in step 3.

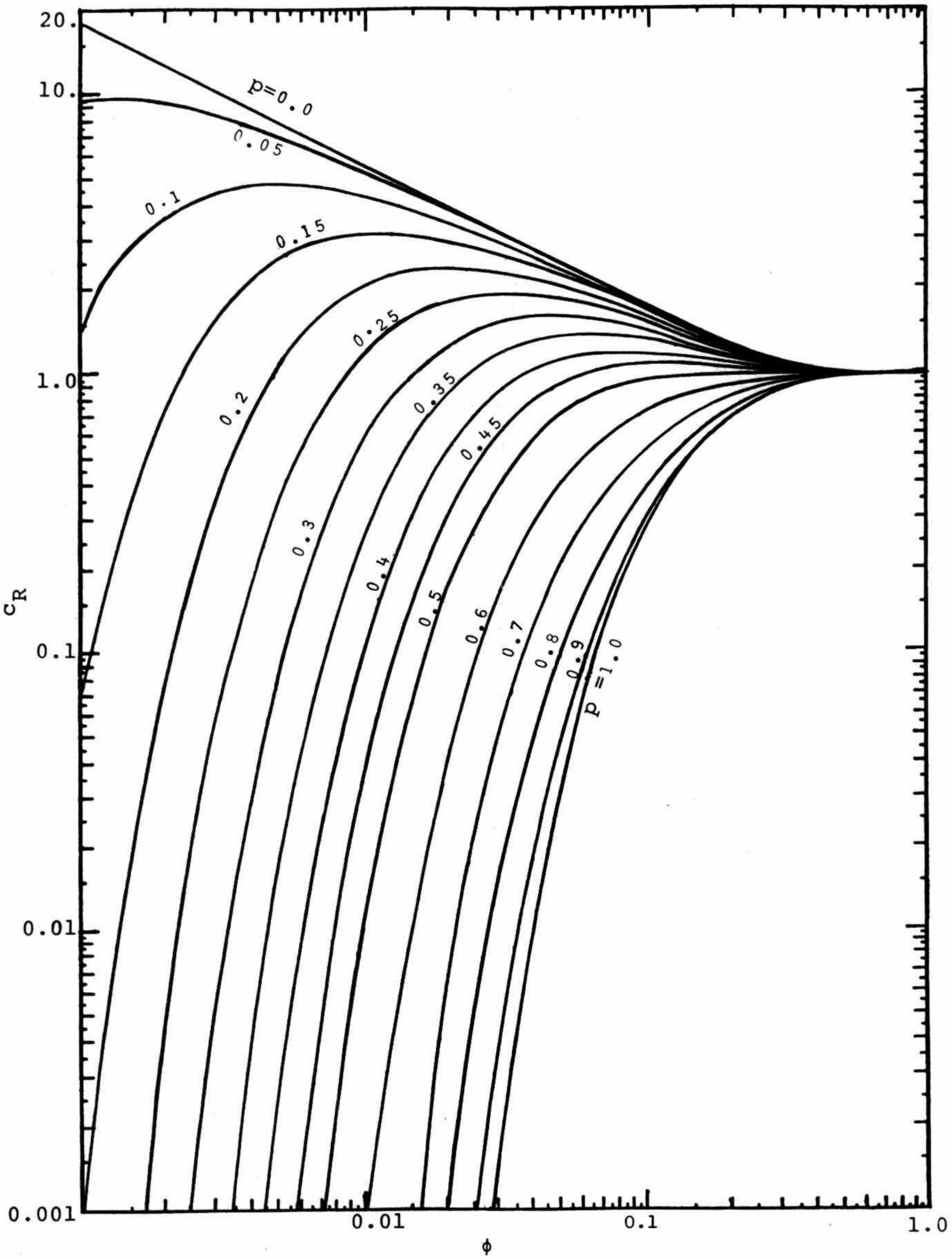


FIGURE 2. DIMENSIONLESS CONCENTRATION DISTRIBUTION IN MIXING ZONE

5. Calculate the concentration of conservative material at each point (x,q) from: $c_c\ (x,q) = c_R\ c_a$.

6. To determine the concentration of nonconservative material, use the equation $c(x,q) = c_c(x,q)\ \exp(-k_{av}x/U)$.

The foregoing graphical procedure should be used in cases where access to more efficient computational procedures is limited and when approximate estimations are acceptable.

The conditions under which the predicted concentrations may be higher than the effluent concentration, c_e, can also be examined with the help of Fig. 2. By definition, $c_R = c_c/c_a = c_cQ/c_eQ_e$; then $c_c/c_e = c_RQ_e/Q$. Therefore, $(c_c/c_e) > 1$ when $(c_RQ_e/Q) > 1$; or $c_c > c_e$ when $c_R > (Q/Q_e)$. The value of c_R can be obtained from Fig. 2 by following steps 1 to 4 outlined above. A comparison of this value with the ratio (Q/Q_e) will indicate whether c_c is greater than c_e or not at a point (x, q). Note that this analysis is valid for conservative materials only. Since the points at which $c_c > c_e$ are generally very close to the outfall, the decay of nonconservative pollutants may not be significant enough to affect the results of this analysis.

2.4 Limited Use Zone Boundary and Allowable Effluent Concentration

The boundary of a LUZ in a shallow river is generally identified by lateral and longitudinal co-ordinates with respect to the outfall. Usually, the lateral boundary of a LUZ is limited to the range 0.2 to 0.4 times the river discharge, Q. The cumulative partial discharge between the outfall bank and the accepted lateral boundary will be denoted by the nondimensional parameter, $p_L = q_L/Q$. A specified pollutant concentration criterion, c_s, must be met within q_L. For a given set of values of c_e, Q_e, q_L and Q, the longitudinal distribution, $c(x_i,\ q_L)$, predicted by Eqs. 16 and 19 attains a maximum value at some x_i and then follows a decreasing trend as shown in Fig. 3. The point at which the concentration attains the maximum value is termed the "critical

point". The longitudinal distance between the outfall and the critical point is termed the "critical distance", x_L, and the maximum concentration is, termed the "critical concentration", c_L. Methods for the computation of x_L and c_L are presented later in Sections 2.5 and 2.6.

Knowing c_L and c_s, the allowable effluent concentration, c_{eA}, can be calculated from the following expression:

$$c_{eA} = c_e \, c_s / c_L \tag{20}$$

Eq. 20 follows from the fact that a change in c_e in Eqs. 16 and 19 would result in a proportional change in c_L, provided all other parameters are held constant; the derivation of Eq. 20 has been presented in a previous publication (11).

The maximum longitudinal boundary of LUZ, x_s, occurs along the discharge shoreline in the case of bank outfalls as shown in Fig. 1 (see Section 1.1). Methods of determining the maximum spreads resulting from c_e and c_{eA} are outlined in Section 2.6.

2.5 Critical Point Method

The development of this method has been described previously including derivation of expressions for critical co-ordinates, graphical method, design procedure and sensitivity analysis (11). However, the method was valid only for a channel of constant width since the derivations were based on width as the lateral co-ordinate. The following subsections describe the salient features of the critical point method based on the stream tube concept, utilizing the partial cumulative discharge as the lateral variable.

2.5.1 Derivation of Expressions:

In order to derive expressions for the critical point occurring along the longitudinal drawn through $q=q_L$, Eq. 19 is simplified by

assuming that k_d is constant in the study stretch and neglecting the effect of images. The simplified equation can be written in the form:

$$c = c_a \sqrt{\frac{Q^2}{\pi D_y x}} \exp\left(- \frac{k_d x}{u} - \frac{q_L^2}{4D_y x}\right) \qquad (21)$$

The limitations imposed by the assumption of n=0, are outlined in a subsection later. It is also assumed that the concentration of the pollutant upstream of the outfall is zero. (A method of accounting for the effect of background concentrations is presented in Section 2.7.)

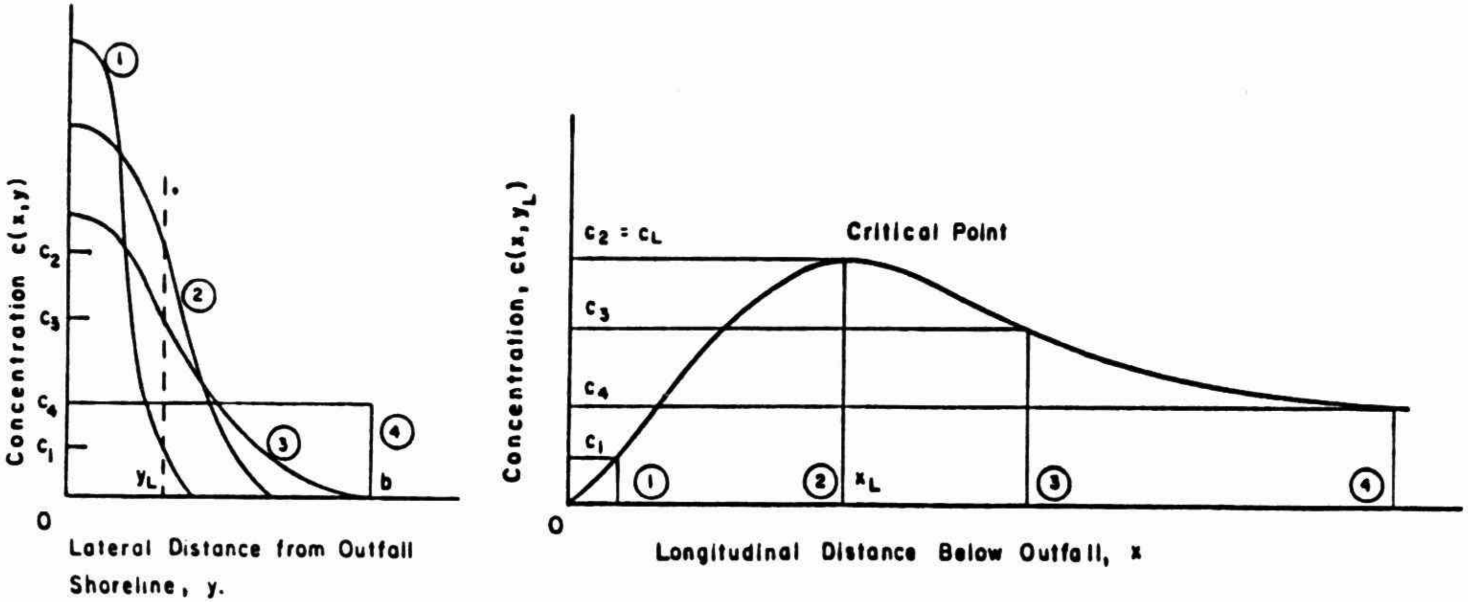
At the critical point shown in Fig. 3, the gradient dc/dx=0 for the concentration to be a maximum. Thus, setting the first derivative of Eq. 21 to zero and solving the resulting quadratic equation for x_L as outlined in a previous publication (11), we get the following relation for the critical distance:

$$x_L = \frac{u}{4k_d} \left\{ -1 + \sqrt{1 + 4 \, (q^2_L k_d/D_y u)} \right\} \qquad (22)$$

The critical concentration, c_L, is obtained by substituting the value of x_L from Eq. 22 in Eq. 21. The allowable effluent concentration, c_{eA} can then be calculated from Eq. 20.

For conservative materials, expressions for x_L and c_L can be derived by setting k_d = 0 in Eq. 21, and following the procedure described above. The resulting expressions are $x_L = bq_L^2/2Q^2 \beta$ and $c_L = c_a Q/2.07 q_L$. The latter expession indicates that the critical concentration of a conservative material is the same at all values of x_L.

The expressions for the critical point, presented above, are based on the assumption that the channel width, b, and mean velocity, u, remain constant in the study stretch of the stream. If the variabilities in b and u are significant, then they should be



(a) CONCENTRATION PROFILES AT CROSS-SECTIONS ① TO ④

(b) LONGITUDINAL PROFILE ALONG A LATERAL BOUNDARY OF LIMITED USE ZONE.

FIGURE 3. CRITICAL POINT OF LUZ

replaced by the moving average values B and U in the above formulations, where

$$B = \frac{1}{2} \frac{\Sigma (b_i + b_{i+1})(x_{i+1} - x_i)}{x_{i+1}} \tag{23a}$$

$$H = \frac{1}{4} \frac{\Sigma (h_i + h_{i+1}) (b_i + b_{i+1})(x_{i+1} - x_i)}{B x_{i+1}} \tag{23b}$$

$$U = Q/BH \tag{24}$$

Some of the features of the critical point method are incorporated in the computer program MIXAPPLN, described in Section 2.10.

2.5.2 Graphical Method:

The expressions for the co-ordinates of the critical point given by Eqs. 21 and 22 can be written in dimensionless terms as follows:

$$G_L = 0.25 \left(-1 + \sqrt{1 + 4 p_L^2 E_L}\right), \tag{25}$$

$$c_r = \sqrt{\frac{E_L}{\pi G_L}} \exp\left(- G_L - \frac{E_L p_L^2}{4 G_L}\right) , \tag{26}$$

where the dimensionless terms are defined by:

$$c_r = c_L/c_a; \quad p_L = q_L/Q; \quad G_L = k_d x_L/u;$$

$$E_L = Q^2 k_d/D_y u; \quad \phi_L = G_L/E_L \tag{27}$$

From Eqs. 13, 14b and 27, we can get the following relations:

$$E_L = k_d b/ \beta u; \quad \phi_L = \beta x_L/b \tag{28}$$

When k_d, b and u are reach-dependent, it is preferable to use their weighted mean values as outlined earlier.

For the range of values of b, u, β and k_d normally encountered in practice, values of E_L are in the range of 0.1 to 1000. The values of G_L were computed from Eq. 25 for various values of E_L and p_L. Figure 4 shows plots of G_L vs. E_L for various values of p_L ranging from 0.1 to 0.6. Values of c_r were then computed from Eq. 26 by using the values of G_L predicted from Eq. 25 for the given values of E_L and p_L. Plots of c_r versus E_L are also shown in Fig. 4 for p_L = 0.1 to 0.6. The family of curves shown on this graph can be used to estimate x_L and c_L in conjunction with the dimensionless terms defined in Eqs. 27 and 28. If the values of E_L lie outside the range shown on Fig. 4 then Eqs. 22 and 21 should be used to calculate x_L and c_L, respectively. A detailed procedure for using Fig. 4 in design computations is presented later. Since the graphical procedure is based on the assumption of constancy of instream process parameters, this method is recommended for use in preliminary planning stages. The iterative procedure described in Section 2.6 should be preferred in cases where the instream process parameters are reach-dependent.

Eqs. 25 and 26 can also be expressed in the following forms by incorporating ϕ_L defined in Eq. 27:

$$G_L = 0.25(-1 + \sqrt{1 + 4\, G_L\, p_L^2/\phi_L}) \tag{29}$$

$$c_r = \sqrt{\frac{1}{\pi\, \phi_L}}\ \exp\left(-\, G_L - \frac{p_L^2}{4\, \phi_L}\right) \tag{30}$$

These two expressions are particularly suitable for the computations involved in the iteration procedure described in Section 2.6.

2.5.3 Effects of Neglecting Side-Wall Reflection of Material:

As stated previously, the critical co-ordinates predicted by Eqs. 21 and 22 are subject to some limitations due to the assumption of n=0. The limitations imposed by this assumption were presented in detail in a previous publication (11). A brief description of the procedure is given below.

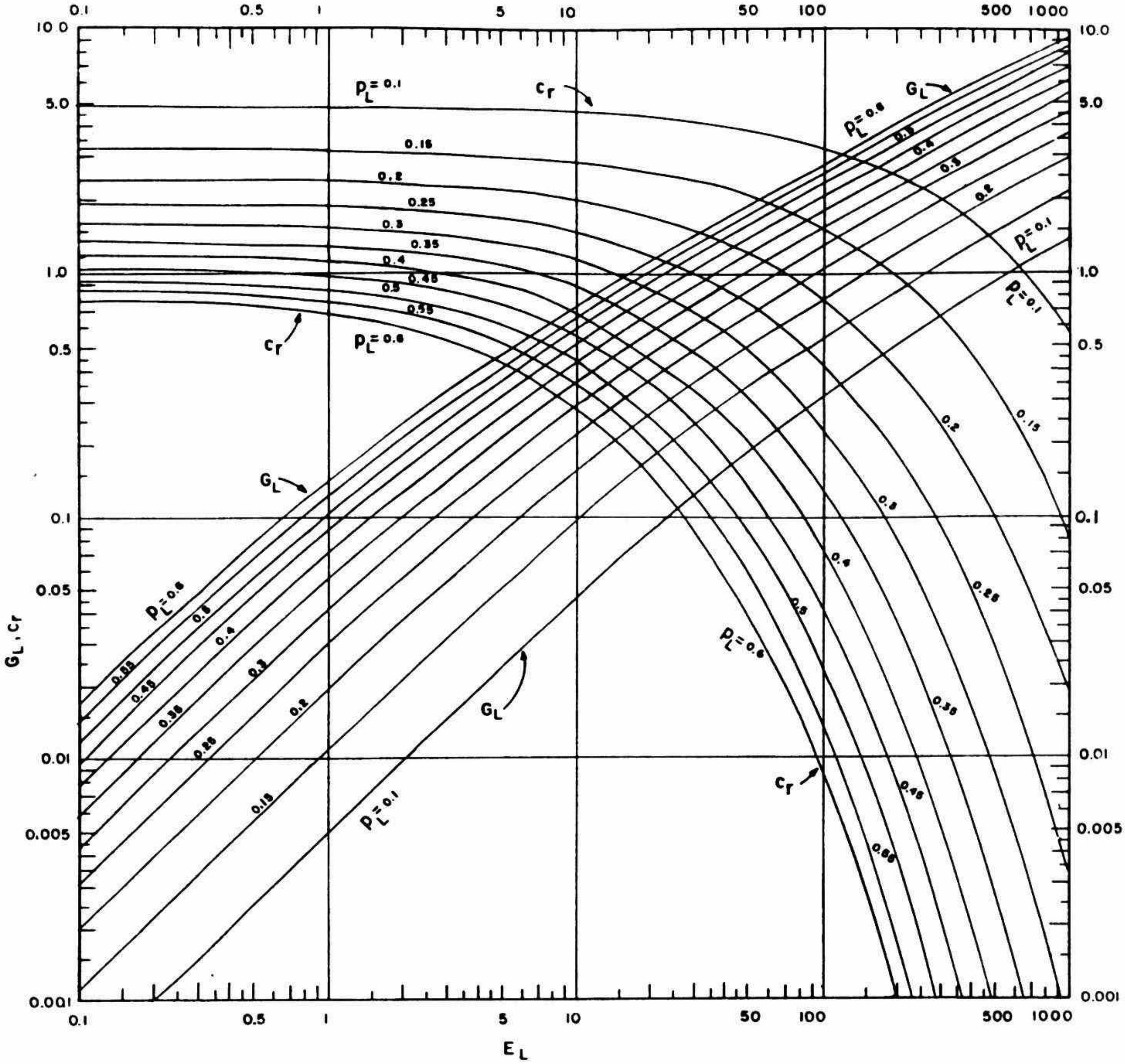


FIGURE 4. NON-DIMENSIONAL CO-ORDINATES OF CRITICAL POINT

The total number of images at a point (x, q) is dependent on ϕ and p as seen from Eq. 18 (with $p_s = 0$). Thus, the number of terms of an exponential series at which the series can be truncated will vary at each point (x, q). By trial and error, it was found that δ is not a constant at all points, but varies in the range of $0 < \delta \leq 1.0$. Initial predictions showed that the dimensionless concentration values $c_R(\phi, p)$ for $\phi \geq 1.0$ remain the same (i.e., concentration distributions are stable) and do not fluctuate when δ is less than or equal to 10^{-4}; the values of c_R at $\phi \geq 1.0$ were equal to unity at all values of p, as stated earlier in Section 2.3.

The transverse distributions of c_R were predicted from Eq. 19 for several values of ϕ, using $\delta = 10^{-4}$. Next, for each value of ϕ the maximum of value of δ which results in the prediction of the distribution of c_R that is in agreement with the distribution predicted with $\delta = 10^{-4}$ was determined by trial and error (by using values of δ equal to multiples of five). The maximum values of δ ranged from 0.00085 to 1.0 at various values of ϕ in the range 1.0 to 0.001, respectively. (Note that stable concentration distributions can be obtained at all values of with $\delta = 0.00085$). From these predictions, the maximum dimensionless value, p_m, at which the total number of images was zero, was determined for each value of ϕ. Using these results a graph of log ϕ versus p_m was prepared (Fig. 5); the values of c_R at all points which lie under the curve are not likely to be affected by the side-wall reflections.

By making use of Fig. 5 and the values of p_L, E_L and G_L, it is possible to examine whether the concentration at the critical point is affected by the side-wall reflections or not, as follows: From Fig. 5, we can obtain the value of ϕ corresponding to $p_m = p_L$; the value of ϕ_L can be calculated from Eq. 27. If ϕ_L is less than ϕ, then the effects of side-wall reflections at the critical point are negligible, whereas ϕ_L greater than ϕ indicates that there may be some approximations in the estimation of c_L and c_{eA}. In such cases, it is possible to predict c_L and c_{eA} by making use of Eq. 16 or Eq. 19, as described earlier.

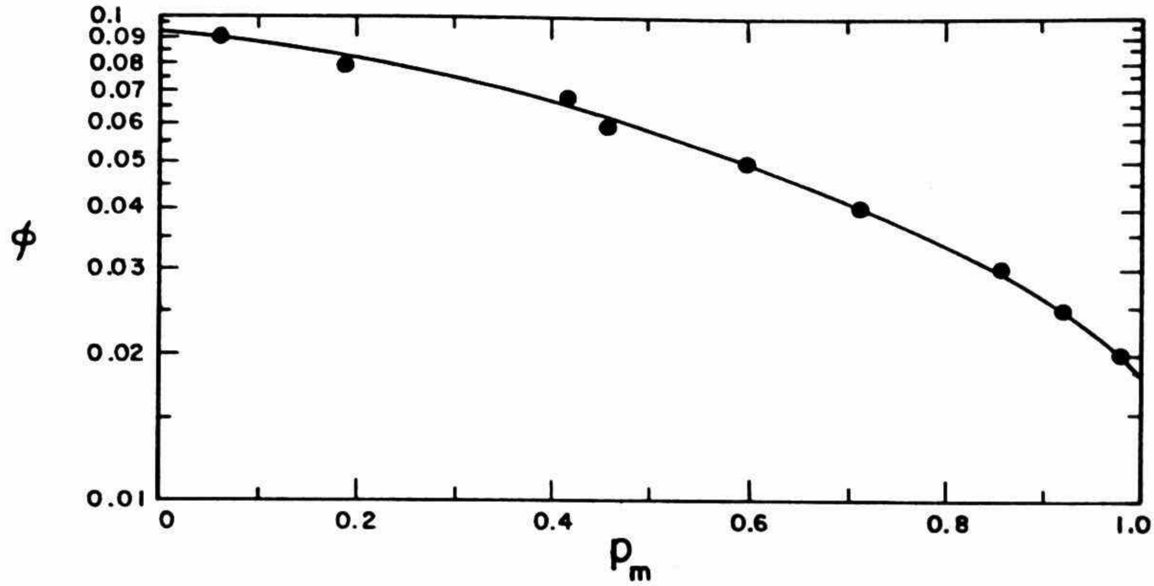


FIGURE 5. SIDE-WALL REFLECTION EFFECTS

2.5.4 Design Procedure for Critical Point Method:

The step-by-step design procedure involved in the critical point method is as follows:

1. Select parameters for the design condition, viz., Q, Q_e, c_e, temperature and pH.

2. Determine the hydraulic parameters b, u (or B, U) and β.

3. Select k_d and apply temperature correction, if necessary.

4. Determine allowable instream concentration, c_s, in accordance with the accepted criterion.

5. Calculate c_a from Eq. 6.

6. Select the lateral boundary of LUZ, q_L, and calculate p_L from Eq. 27. Obtain the value of ϕ corresponding to $p_m = p_L$ from Fig. 5.

7. Calculate E_L from Eq. 27.

8. From Fig. 4, determine the values of G_L and c_r corresponding to E_L and p_L obtained in steps 6 and 7. If the values of E_L and p_L are outside the range shown in Fig. 4, use Eqs. 21 and 22 to compute x_L and c_L, and proceed to step 10.

9. Calculate the co-ordinates of the critical point, x_L and c_L, from $x_L = G_L u/k_d$ and $c_L = c_r c_a$ (obtained from Eq.27).

10. Calculate the value of ϕ_L from Eq. 27. If this value is less than or equal to the value of ϕ obtained in step 6, then c_L is not affected by the side-wall reflection; otherwise an approximation may be involved in the estimation of c_{eA} from Eq. 20.

11. If c_L is not equal to c_s, then calculate c_{eA} from Eq. 20. This value represents the allowable effluent concentration based on the LUZ concept. If $c_L = c_s$, then $c_e = c_{eA}$. Compute percent treatment = 100 $(c_e - c_{eA})/c_e$, when c_{eA} is less than c_e.

2.6 Iterative Procedure for Allowable Effluent Concentration and Longitudinal Spread

The iterative procedure is based on a combination of Eq. 16 (or Eq. 19) and the dimensionless expressions for the co-ordinates of the critical point given by Eqs. 29 and 30. This procedure is included in the computer program MIXAPPLN described in Section 2.10. Fig. 6 shows the salient features of the iterative procedure graphically. The computational details are described in the following subsections.

2.6.1 Allowable Effluent Concentration:

In the MIXAPPLN program, the concentration predictions of Eq. 16 are used to search for the transect at which the concentration is maximum for a given q_L. Using this concentration, c_{crit}, and the corresponding distance, x_{crit}, new values of x_L and c_L are calculated from Eqs. 29 and 30 using weighted mean values of decay rate, width and velocity as first estimates (determined from Eqs. 9, 23 and 24). If the absolute percentage difference given by $\{ 100 (x_L - x_{crit})/x_L \}$ is greater than an arbitrary criterion of 5%, the procedure is repeated until the difference is within 5%. The values of x_L and c_L determined in the final iterative step are printed out along with the corresponding c_{eA} values computed from Eq. 20.

2.6.2 Maximum Longitudinal Spread:

In the case of discharge from a bank outfall, the maximum longitudinal spread, x_s, occurs along q=0, as stated before (in Section 2.4). Then, the spread, x_s, resulting from c_e is obtained by drawing a horizontal line through c_s as indicated on

Fig. 6. An effluent concentration, c_{eA}, determined by Eq. 20 would result in $c_L = c_S$ for given q_L. The corresponding concentrations along q=0 would also change proportionately since all other parameters remain unchanged; the value of x_s corresponding to c_{eA} is obtained as before. Note that the latter value for x_s can also be obtained by drawing a horizontal line through c_L and a vertical through the intersecting point, thus eliminating the need to draw the longitudinal concentration distribution, c(x, o), corresponding to the allowable effluent concentration, c_{eA}. The development of an iterative method for computing x_s, thus eliminating the need for the graphical procedure, is described below.

The transect at which c(x, o) is just greater than or equal to c_s is determined by searching the predictions of Eq. 16 resulting from a given effluent concentration, c_e, and other input parameters. The corresponding values of distance and concentration are denoted by x_{est} and c_{xs}. When the absolute percentage difference, $100(c_{xs} - c_s)/c_s$, is less than or equal to an arbitrarily specified criterion of 5%, the maximum spread, x_s, is set equal to x_{est}. If the absolute difference exceeds 5%, a new value of the distance, x_s, is estimated from $x_s = x_{est}(1.0 + f_x)$, in which $f_x = (dx/x)$ is a factor denoting a relative change in x. (An expression for f_x is derived later.) Using the latter value of the distance, x_s, a new concentration value, c_{xs}, is calculated from Eq. 16 and compared with c_s as outlined above. The iterative procedure is repeated until the absolute difference is within the specified limit of 5%. The maximum shoreline distance, x_{sce}, at which the concentration is very close to c_s, is then taken equal to x_s obtained in the final iteration step. If the shoreline concentration c(x, o) at a defined downstream boundary exceeds c_s, or if the number of iterations is greater than 30, the value of x_{sce} is set equal to a negative number.

The derivation of an expression for f_x is based on the method of determining relative error or relative change (described in standard

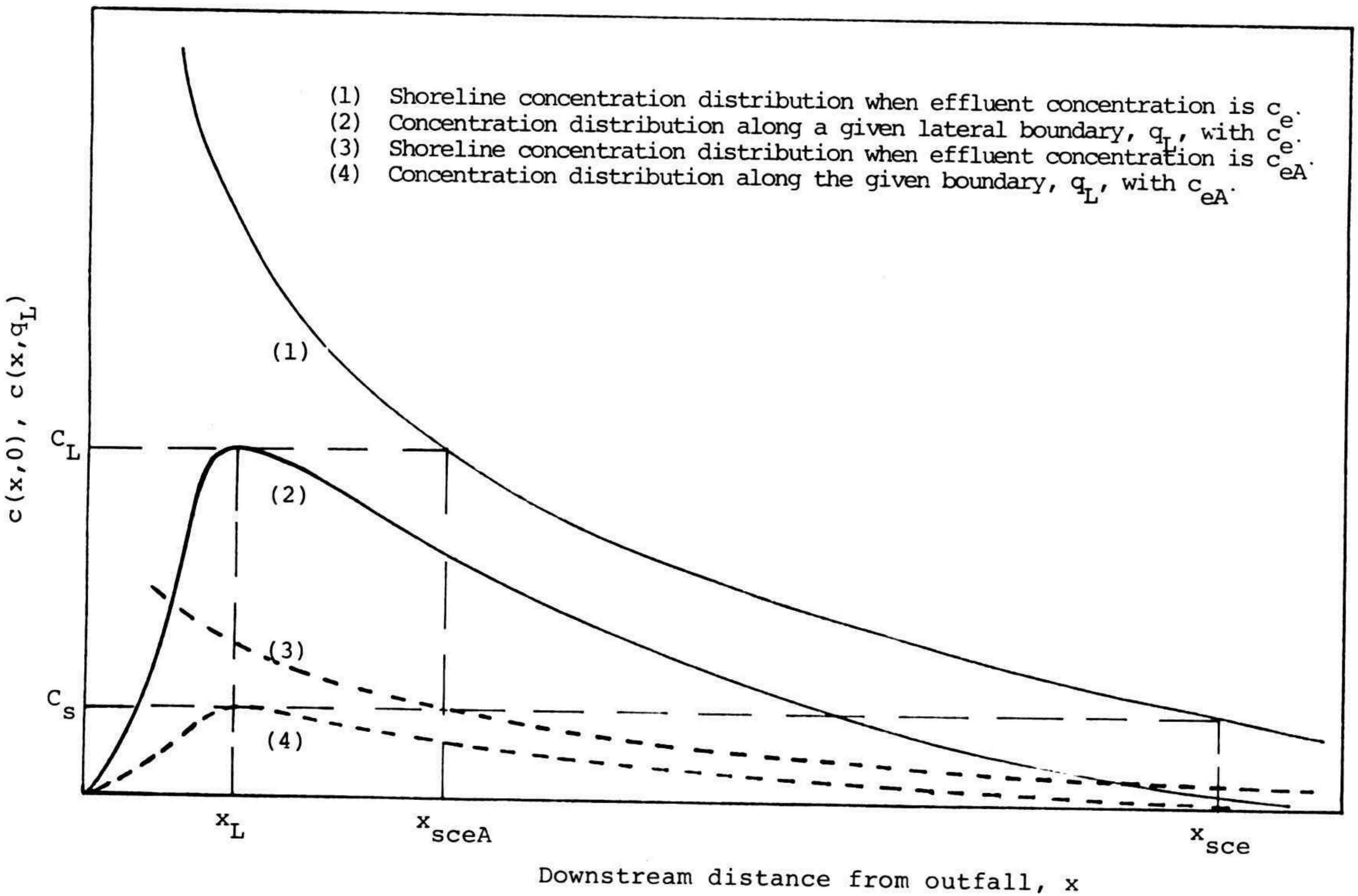


FIGURE 6. GRAPHICAL REPRESENTATION OF ITERATIVE PROCEDURE

text books on calculus). An approximate form of Eq. 19 is used in this derivation procedure. Neglecting the effect of images and assuming that k_d is constant in the study stretch, Eq. 19 simplifies to:

$$c(x, o) = c_a \sqrt{\frac{b}{\pi \beta x}} \exp(-k_d x/u) \tag{31}$$

Taking logarithms on both sides, differentiating with respect to x and simplifying, we get the expression

$$\frac{dx}{x} = - \frac{1}{1 + (k_d x/u)} \frac{dc}{c} \tag{32}$$

In Eq. 32, the term on the left represents the relative change in x. The term (dc/c) denotes the relative change in c, and is set equal to $\{(c_{xs} - c_s)/c_s\}$; then the factor $f_x = (dx/x)$ is computed from Eq. 32. Note that f_x will be negative when (dc/c) is positive and vice versa. In each iteration, the computations make use of the moving average values, k_{av}, B and U, which are calculated from Eqs. 9, 23 and 24.

When the allowable effluent concentration, c_{eA}, is determined for a given q_L as outlined above, the associated critical concentration, c_L, is used in place of c_s to compute the longitudinal spread as stated earlier. The maximum spread in these cases is denoted by x_{sceA}.

2.7 Effects of Background Concentrations

The foregoing formulations have been based on the assumption that the concentration of the nonconservative pollutant of concern in the river just upstream of the outfall is zero. When the background concentrations are not negligible, it is possible to include their effects by assuming that the background concentration distribution in the cross section of the river is uniform throughout the study

stretch. This assumption is valid in cases where the effect of upstream discharge of pollutants results in a uniform cross-sectional distribution just upstream of the effluent discharge under consideration. Generally, the decay rate of background pollutants, k_b, would be somewhat different from those associated with the effluent discharge, being smaller in most cases.

The computer program MIXAPPLN incorporates background effects by calculating the concentration at each transect, c_{bi}, due to an upstream background value, c_b, from the relation

$$c_{bi} = c_b \exp(-k_{bT} x_i / \bar{u}_i) \tag{33}$$

in which $\bar{u}_i$ = average value of velocity from the outfall to the transect i; and k_{bT} = temperature-corrected value of k_b. The value, c_{bi}, is then added to each concentration value, c(x, q), to give the total concentration resulting from the upstream and effluent discharges of the pollutant. All computations pertaining to concentration distributions, critical co-ordinates and maximum longitudinal spread include the background effects.

When there are a series of outfalls discharging into the same river, the effect of an upstream discharge just above the next downstream outfall would depend on the distance between the two outfalls, x_w, as well as on the characteristics of the waste constituent and the hydraulic characteristics of the river channel. When the oufalls are sufficiently far apart, the cross-sectional concentration distribution would likely approach uniformity. In such cases, the one-dimensional assumption could be used to estimate concentrations in the river just above the downstream outfall locations so that these values can be used as background levels in modelling the latter discharge. These computations are performed by using an average decay rate, k_{av} (the weighted mean value computed from the data input to the MIXAPPLN program). Then, the concentration, c_{wa}, at the downstream boundary due to the effluent discharge is computed from:

$$c_{wa} = c_a \exp(-k_{av}\, x_w / U) \tag{34}$$

The contribution of the background concentration is calculated from Eq. 33 with $x_i = x_w$. The sum of these two values gives the downstream boundary concentration. Thus Eqs. 33 and 34 can be used to examine the effect of an upstream pollutant discharge on the instream background level at a downstream location of interest such as a water supply intake or another effluent outfall location.

2.8 Mixing Zone Length

As shown in Fig. 2, the cross sectional concentration profile of a conservative material attains uniformity at $\phi = 1.0$, thus defining the theoretical mixing zone length. Since $\phi = (\beta x/B)$, the mixing zone length, x_{mz}, is given by:

$$x_{mz} = B/\beta \qquad (35)$$

The weighted mean value, B, is used instead of the local width to get a more accurate estimate of x_{mz}. Some expressions of practical interest for x_{mz} are presented by Holley, et al (13). In the computer program MIXAPPLN, Eq. 35 is used to calculate x_{mz}. In the case of nonconservative pollutants, Eq. 35 is likely to overestimate x_{mz} since the decay effect is not included.

The cross sectional average concentration of a pollutant of concern at the distance, x_{mz}, below the outfall is given by the equation

$$c_{mz} = c_a \exp(-k_{av}\, x_{mz}/U) \qquad (36)$$

2.9 Scale-up of Model Parameters

When the model is applied to predict pollutant concentrations in mixing zones for specified management options such as treatment plant expansion, flow augmentation, seasonal variabilities in streamflow and temperature, etc., it is necessary to scale up the model parameters to account for the variabilities associated with a defined option. The relationships commonly used to scale up the parameters affected by changes in design temperature and design streamflow are presented below. (These relationships are utilized in the computer programs MIXCALBN and MIXAPPLN).

2.9.1 Temperature Effects:

The temperature dependence of the decay rates is expressed by the modified van't Hoff-Arrhenius relationship:

$$k_{T2} = k_{T1} \quad \Theta^{T_2 - T_1} \tag{37}$$

in which k_{T1} and k_{T2} = decay rate coefficients at temperatures T_1^oC and T_2^oC, respectively; and Θ = temperature correction factor. The values of Θ for TRC and ammonia may be taken as 1.03 and 1.106, respectively (7, 8). The decay rates may also depend on turbulence and other hydraulic characteristics of stream channel, and effluent and stream water quality; however, only the effect of the primary variable temperature, is considered in this study.

2.9.2 Effect of Change in Streamflow:

A change in river discharge results in corresponding changes in depth, h, velocity, u, and top width of water surface, b. In water quality studies, the changes in the hydraulic parameters are generally expressed by the Leopold-Maddock equations (7)

$$b_1 = b_2 \ (Q_1/Q_2)^{bex} \tag{38}$$

$$h_1 = h_2 \ (Q_1/Q_2)^{hex} \tag{39}$$

$$u_1 = u_2 \ (Q_1/Q_2)^{uex} \tag{40}$$

in which b_m, h_m and u_m represent in order, the mean values of width, depth and velocity at a flow Q_m (m = 1, 2); and bex, hex, and uex represent exponents, such that their sum is equal to unity. The values of the exponents are estimated from the channel geometry and time of travel data.

2.10 Computer Programs

2.10.1 MIXCALBN Program:

The computer program MIXCALBN (MIXing zone - CALiBratioN model), written in FORTRAN language, is useful for model validation studies. The model accepts the survey data as input along with instream process parameters (which are evaluated as outlined in Chapter III). The output of the model is in a matrix form, the lateral distributions at each transect being given in concentration units as well as in relative concentration units defined by $c_r = c(x,q)/c_a$. (Note: The program is set up for execution on a time-sharing terminal).

A listing of the computer program MIXCALBN is given in Appendix A, along with a list of variable names. The program list also incorporates comments indicating brief descriptions of computational details. As an illustration of typical input and output of the program, an example based on the one cited by Verboom (25) was selected. The input data and output matrix table are given in the Appendix, along with the details of the example.

2.10.2 MIXAPPLN Program:

The second computer program MIXAPPLN (MIXing zone - APPLicatioN model), also written in FORTRAN and operational through a time-sharing terminal, has been developed for performing design option computations. The program input includes essentially the same data as the MIXCALBN model, as well as data on options normally considered in design runs (viz., numbers and values of upstream flow rates, effluent flow rates and temperatures, as well as pH values in the case of ammonia).

A listing of the program MIXAPPLN is provided in Appendix A. The list of variables in this program includes those of the MIXCALBN program; the additional variables appearing in the MIXAPPLN program are also included in the list of variable names.

The program MIXAPPLN is set up to handle eight upstream flow rates, six effluent flow rates, and six temperature values, as well as four pH values in the optional case of prediction of toxic ammonia levels. However, these can easily be increased or decreased by changing the size of the appropriate arrays. For each run, the program output consists of a table of transverse concentration distributions at each transect for p_L = 0 to 0.5 and a second table which includes c_L, x_L, c_{eA} and x_{sceA} for p_L = 0.2, 0.3 and 0.4. The mixing zone length, x_{mz} and the corresponding concentration, c_{mz}, are also printed out along with the maximum longitudinal spread, x_{sce}, corresponding to the input effluent concentration, c_e.

Some of the results of each run are stored in arrays and printed out as a summary table. A typical input/output of the program MIXAPPLN is presented in Appendix A. (Note: When the pollutant is not ammonia, the pH values are not input to the program; however, the program sets pH = 7.0, this value being printed as part of the output.)

III DATA COLLECTION AND ANALYSIS PROCEDURES

3.1 Field Survey Procedures

The field surveys are designed to collect data on outfall discharge and background water quality characteristics, and on the transverse distribution of various parameters of interest at four to six transects. The details of field surveys are usually dependent on site-specific conditions, as outlined in the case studies in Chapters IV and V. A general field study procedure is described below.

The location of transects can be based on preliminary *in situ* measurements of a conservative parameter (e.g. conductivity) at selected access points to establish the approximate longitudinal boundary of the mixing zone. The selected locations of transects must be marked on a map so that they can be readily identified in the field during further surveys. Cross-sectional depths must be measured at a minimum of 15 points at known lateral distances measured from a reference bank (usually the outfall bank). Velocity measurements must also be taken at least at two transects by following standard streamflow gauging procedures; however measurements at all transects are desirable. (Note: At transects where velocities are not measured, the Manning's equation can be used to simulate the velocity profiles using measured depth profiles as outlined later in Section 3.2).

Water samples must be collected at the upstream boundary and at the effluent outfall, and at each point where cross-sectional depths and velocities are measured. However, the samples can be collected at selected points at each transect (viz., less points outside effluent plume, alternate points, etc.) to reduce sample analysis costs, in which case the concentrations at other points are obtained by interpolation during data analysis. In order to account for the possible effects of fluctuations in effluent water quality and discharge on the instream concentrations, the sampling is carried out either by following the same plug of water beginning at the

outfall and proceeding to successive downstream transects, or during a round-the-clock intensive survey when samples are collected at each point at 3 to 4 hr. intervals. Obviously, the selection of a sampling methodology would depend on the man-power, time and other resource constraints, as well as the objectives of the study.

In situ measurements of temperature, pH and conductivity must be taken along with collection of samples which are to be analysed in the laboratory for nonconservative pollutants of concern. In some cases, it may be desirable to inject a solution of dye continuously to gather data on the transverse distribution characteristics of the river. This is particularly useful to simulate effluent discharge from proposed outfall locations, and in cases where relocation of an existing outfall is being considered. The dye injection must be maintained for 2 to 3 hours or more to establish steady state conditions. The cross-sectional distributions of dye at selected transects can be obtained directly through fluorometric tracing; however, in shallow rivers where the passage of a boat is difficult, it will be necessary to collect water samples at known cross-sectional points for dye concentration analysis.

Generally, two surveys must be carried out under different instream hydraulic conditions so that the data of one survey can be used to calibrate the model, which is then verified using the data of the second survey. If man-power and other resource constraints are limited, then the data of one survey may be used to validate the model. The limitations of the latter approach must be given due consideration in using the predictions of the model for planning and management purposes.

3.2 Data Analysis Procedure

The data collected during one of the field surveys are utilized to determine the parameters required for modelling. A computer program MIXANDAT, written in FORTRAN language, is utilized to perform the following computations using the survey data as input:

1. Average depth and velocity at each transect.
2. Simulation of velocity distributions at cross-sections where velocities are not measured.
3. Shape-velocity factor at each transect.
4. Mass flux values of conservative and nonconservative materials at each transect.
5. Variance of cross-sectional distributions of conservative materials (required to evaluate the dispersion characteristics).

The computations involving mass flux and variance utilize net concentration values of a material determined from the measured values minus background value of that material in river water. If a net concentration is negative, then it is set equal to zero. (Note: The variance values computed from the distributions of non-conservative material have not been used in evaluating dispersion coefficients).

All calculations in the MIXANDT program are based on the trapezoidal approximation of the integrand. A listing of the computer program is presented in Appendix B, along with a list of the variable names. Details of the various computations are presented below. For simplicity, the subscript, i, associated with the transects will be dropped from the relationships, except where it is necessary to retain it for clarity.

3.2.1 Mean Depth and Velocity:

At a given transect, i, the cross-sectional average values of depth, $\bar{h}$, and velocity, $\bar{u}$, are computed from:

$$\bar{h} = A/b \tag{41}$$

$$\bar{u} = Q/A \tag{42}$$

in which A = cross-sectional area of flow; and b = top width of water surface at the transect, i. The cross-sectional area is computed from:

$$A = \sum_{k=1}^{m} 0.5(h_k + h_{k+1})(y_{k+1} - y_k) \quad (43)$$

in which m = number of vertical strips in the cross-section; and h_k = local depth at a lateral distance, y_k, from the reference bank of the river. The total discharge in the river, Q, is given by the sum of upstream and effluent flow rates.

3.2.2 Transverse Distribution of Flow:

The cross-sectional distribution of streamflow at a given transect is calculated from

$$q_k = 0.25(h_k + h_{k+1})(u_k + u_{k+1})(y_{k+1} - y_k), \quad (44a)$$

$$Q'_j = \sum_{k=1}^{j} q_k, \quad (44b)$$

in which q_k = discharge through an elemental strip, k; Q'_j = partial cumulative discharge between the reference bank and j^{th} strip; and u_k = depth-averaged local velocity at a lateral distance, y_k, from the reference bank. Measured velocity values are used in Eq. 44a whenever such data are available. At transects where depth profiles are available and velocity measurements are not taken, the velocity distributions are synthesized by making use of a resistance equation (either Manning's or Chezy's equation) following Pailey and Sayre (20), as outlined below.

In a shallow, wide river the local velocity u, at a point where the depth is h, can be expressed by the Manning's equation

$$u = \frac{1}{n'} h^{2/3} s_e^{1/2} \quad (45a)$$

in which n' = the Manning's roughness coefficient; and s_e = energy gradient (approximated by the slope of the channel bed surface). Similarly, the cross-sectional mean velocity is given by:

$$\bar{u} = \frac{1}{n'} \bar{h}^{2/3} s_e^{1/2} \tag{45b}$$

From Eqs. 45a and 45b we get:

$$(u/\bar{u}) = (h/\bar{h})^{2/3} \tag{46a}$$

This expression can be generalized to account for possible variation of bed roughness in a given stretch, as follows:

$$(u/\bar{u}) = f_1 \ (h/\bar{h})^{f_2}, \tag{46b}$$

in which f_1 and f_2 are empirical coefficients. According to Manning's equaion $f_1 = 1.0$ and $f_2 = 2/3$; when Chezy's equation is used $f_1 = 1.0$ and $f_2 = 1/2$. The values of f_1 and f_2 can be determined by fitting the velocity distribution data collected at a few transects to Eq. 46b. In the program MIXANDAT, the default values for f_1 and f_2 are 1.0 and 0.67, respectively.

Eq. 46b is used to simulate the velocity profile at each transect where velocities are not measured (provided depth profiles are available). The simulated or measured velocities are used to compute a value of total discharge Q', at a given transect using Eq. 44b. If Q' differs from the known total discharge, Q, the velocities are multiplied by a correction factor, (Q/Q'), to obtain a new set of simulated values at each transect, and the latter values are used in further computations.

3.2.3 Shape-Velocity Factor:

The following expression developed by Beltaos (1, 2) is used to calculate the shape-velocity factor, ψ, at each transect.

$$\psi = \int_0^1 (h/\bar{h})^3 \ (u/\bar{u})^2 \ d(y/b), \tag{47}$$

The value of ψ is calculated by using the following summation expression:

$$\psi = \frac{1}{32 b\bar{h}^3 \bar{u}^2} \sum_{k=1}^{m} (h_k + h_{k+1})^3 (u_k + u_{k+1})^2 (y_{k+1} - y_k) \qquad (48)$$

in which u_k = measured or simulated local mean velocity at a distance, y_k, from the reference bank. Results presented by Beltaos (2) show that the shape-velocity factor varies in the range 1.0 to 3.2.

3.2.4 Mass Flux and Relative Concentrations:

The mass flux, f_k, of each material (conservative or nonconservative) passing through a vertical strip, k, at a given transect, i, is determined from the relation

$$f_k = (1/8)(y_{k+1} - y_k)(h_k + h_{k+1})(u_k + u_{k+1})(c_k + c_{k+1}), \qquad (49)$$

in which c_k represents the measured concentration at y_k, minus background value. The total net flux, F_i, of each material at the transect, i, is then determined from

$$F_i = \sum_{k=1}^{m} f_k \qquad (50)$$

The total gross flux of a conservative material at each transect (equal to total net flux plus background flux) must be equal to the sum of effluent and upstream flux values. However, some deviations generally occur in practice due to various reasons which are site-specific. The cross-sectional average concentration, c_{atr}, of each material is computed from $c_{atr} = F_i/Q$. Nondimensional (i.e., relative) concentration values of conservative materials are then calculated from $c_{ra} = c_k/c_{atr}$; these values will be used for comparison with the predictions of the stream tube model. The

use of relative concentration values removes, to some extent, the effects of deviations in the mass flux values of conservative materials at various transects, thus making c_{ra} values better suited for comparative purposes during model validation studies.

The total gross mass flux values of a nonconservative material at various transects are used to determine the first-order decay rate of the material by preparing a semi-log plot of mass flux versus time of travel. (Note: Sometimes, distance is used in plotting instead of time of travel, in which case mean velocities are also required to compute the decay rates.)

The units of the mass flux values determined from Eq. 49 and 50 are dependent on the units of various quantities appearing on the right hand side of Eq. 49. Thus, if length is in meters, time in seconds and concentration in ug/L, then the units of mass flux will be 1000 ug per second or mg/s (since 1 m^3 = 1000 L); if the concentrations are in mg/L, then the mass flux units are gm/s.

3.2.5 Variance of Tracer Distributions:

The growth of variance of a tracer distribution at successive transects below a source is indicative of the dispersion characteristics of the stream stretch. Traditionally, the variance, σ_y^2, of the c(x,y) versus y distribution, has been related to the transverse dispersion coefficient, e_y, on the assumption that the stream channel cross-section approximates a rectangular channel; hence, it is subject to limitations for application to natural stream channels. In order to overcome some of those limitations, a generalized change of moment (GCM) method was developed by Holley, Abraham and Siemans (13). In the formulation of the streamtube model, Yotsukura and Cobb (29) have developed a relation between the diffusion factor, D_y, and the variance, σ_q^2, of c(x,q) versus q distributions. As shown in Section 2.2 the expressions for D_y include e_y; also, both D_y and e_y can be related to bulk flow parameters of channel (see Sections 2.2, 3.3 and 3.4). The variance values, σ_y^2 and σ_q^2,

are thus required to determine the transverse dispersion characteristics; hence, methods of computing σ_y^2 and σ_q^2 are presented below.

$$\sigma_y^2 = \frac{2e_y x}{u} \quad , \tag{51a}$$

$$\sigma_q^2 = 2 D_y x \tag{51b}$$

where u represents the mean velocity of flow in the x-direction, and σ_y^2 and σ_q^2 denote in order, the variance values of c(x,y) vs. y and c(x,q) vs. q distributions at a distance, x, below the source. (Note: The units of σ_y^2 and σ_q^2 are $[L^2]$ and $[L^6 T^{-2}]$, respectively.) The variance values of the distributions may be computed by using the peak concentration values and by the method of moments; brief descriptions of these methods are presented below:

In the peak concentration method, the variance, σ_y^2, is computed from a relation given by Sayre and Chang (22), which can be written in the following form:

$$\sigma_y^2 = \frac{1}{2\pi (c_{max}/A_c)^2} \quad , \tag{52}$$

in which c_{max} = maximum concentration of a tracer at a transect distant, x, below the outfall; and A_c = area under the c-y curve. Eq. 52 gives σ_q^2 when A_c is replaced by the area under the c-q curve, A_q.

In the method of moments, the variance is obtained from the second moment of a tracer distribution curve divided by the area under the curve; an expression for the variance of the c-y distributions is given by:

$$\sigma_y^2 = \frac{\int_o^b y^2 \, c \, dy}{\int_o^b c \, dy} \tag{53a}$$

For computational purposes, the following expression is used:

$$\sigma_y^2 = \frac{1}{4} \frac{\sum_{k=1}^{m} (y_{k+1} + y_k)^2 (c_{k+1} + c_k) (y_{k+1} - y_k)}{\sum_{k=1}^{m} (c_{k+1} + c_k) (y_{k+1} - y_k)} \tag{53b}$$

in which c_k = net concentration at a lateral distance, y_k, from the outfall bank of channel and b = width of channel at a given transect. When dy, y and b are replaced by dq, q and Q, respectively, in Eqs. 53a-b, we get the variance, σ_q^2, of the c-q distributions. The units of σ_y^2 and σ_q^2 will be m^2 and m^6/s^2, respectively, when widths and discharges are expressed in the metric units.

In non-prismatic channels in which transverse convective velocities are present, steady state concentration measurements may be used to determine e_y by using a generalized change of moments (GCM) method as outlined by Holley, Seimons and Abraham (13). This method is applicable to practical situations where data on depth, velocity and concentration profiles are available at closely spaced cross-sections. Neglecting the effects of transverse velocities, a modified method has been developed for determining e_y, by incorporating an important feature of the GCM method (14). In this method, a variance σ_f^2 is determined by using the product (chu) instead of c in Eqs. 53a-b, where h and u represent the local values of depth and velocity, respectively. The product (chu) represents the advective tracer flux per unit width of channel; its cross-sectional distribution is referred to as the "unit flux curve" in this paper. Under steady state conditions, Eq. 51a can be used to determine e_y when either σ_y^2 or σ_f^2 values are known at various values of x.

The advantages and disadvantages of the foregoing methods of computing the variances are as follows:

The peak concentration methods are simpler than the moment methods; since they rely on the value of c_{max} at each transect, utmost care should be taken in the measurement of c_{max}. The computations of the moment methods are more involved, requiring complete cross-sectional distributions. Both methods are based on the assumption that the effects of side-wall reflections on the measured concentration values are negligible, and thus, the results may be approximate.

The computer program MIXANDAT presented in Appendix B, includes computational steps for σ_y^2, σ_f^2 and σ_q^2. All of these values have been utilized to examine various aspects of dispersion characteristics in the case study presented in Chapter IV. (Note: For modelling studies, it is sufficient to utilize σ_q^2 values which are required to estimate β .)

3.3 Transverse Dispersion Coefficient

According to Eq. 51a, a plot of variance, σ_y^2, versus x would result in a straight line of slope $(2e_y/u)$, from which e_y can be determined. This method is particularly valid for prismatic channels. In natural streams, the variabilities in the hydraulic characteristics of a channel affect tracer concentration distributions and variance values at different transects. Thus, the variance versus x plots may not display a steady growth of the variance values. For application to such situations, Eq. 51a can be modified by incorporating the hydraulic parameters which are likely to affect dispersion characteristics. The following subsections describe the relationships between e_y and channel hydraulic parameters, and derivation of modified expressions for the dispersion coefficient.

3.3.1 Relations between Dispersion Coefficient and Hydraulic Parameters:

A review of the literature indicates that e_y can be expressed as a function of the bulk flow parameters of a channel (6, 13, 16). On

the hypothesis that the transverse dispersion process is a function of the hydraulic radius of the channel, r, and the bed shear velocity, u_*, Elder (6) obtained the relation $e_y = \alpha_1 r u_*$, where α_1 is a nondimensional transverse dispersion coefficient. Based on laboratory experimental results, Elder found that $\alpha_1 = 0.23$. In natural streams with straight channel alignment, α_1 is found to vary from 0.25 to 0.75, whereas for meandering streams values in the range 0.3 to 7.2 are reported; investigations under ice-cover conditions reported by Beltaos (1) show values in the range of 0.35 to 1.16. Since a direct measurement of u_* in a natural stream is difficult and laborious and α_1 varies over a wide range, Elder's relation is subject to limitations in practical use.

Holley, Siemons and Abraham (13) expressed e_y as a function of the average values of depth and velocity of flow by the following relationship:

$$e_y = \alpha_2 h\, u \tag{54}$$

where h and u represent mean depth and mean velocity in the study stretch, respectively; and α_2 is a nondimensional transverse dispersion coefficient. The values of α_2 in natural streams, reported by Holley and Abraham (14), are in the range 0.02-0.04.

As stated in Chapter II, Lau and Krishnappan (16) related e_y to the width and velocity of flow; the expression for e_y, developed from a dimensional analysis, is given by:

$$e_y = \beta_e\, b\, u \tag{55}$$

where β_e is a nondimensional coefficient. The values of β_e were found to vary from 2.78×10^{-4} to 15.17×10^{-4}, in rectangular flumes. (Note: In this study, Eq. 55 has been selected for determining the transverse dispersion coefficient, as outlined previously. However, both Eqs. 54 and 55 are used below to develop modified relationships suitable for evaluation of dispersion

coefficients using field data from streams and rivers. In the next chapter, the applicability of the modified relationships will be tested using field data).

3.3.2 Modified Expressions for Dispersion Coefficient:

When the geometrical and hydraulic characteristics vary in a given stretch of a natural stream, it is desirable to divide the stretch into a number of reaches as stated in Chapter II, such that the flow characteristics within each reach are essentially the same, but differ from one reach to another. It is then possible to incorporate the effects of variations in depth and velocity into the relation for e_y; by combining Eqs. 51a and 54 the following modified expression is obtained:

$$\frac{\sigma_y^2}{h_i} = 2\ \alpha_2 x_i, \tag{56}$$

where $\alpha_2 = e_y/h_i u_i$, and h_i and u_i represent, in order, the average values of depth and velocity at a transect, i, distant x_i below the outfall. Similarly, by incorporating the variations in width and velocity the following modified relation is obtained from Eqs. 51a and 55:

$$\frac{\sigma_y^2}{b_i} = 2\ \beta_e x_i \tag{57}$$

where b_i represents the top width of water surface at a transect distant x_i below the source, and $\beta_e = e_y/b_i u_i$. For graphical presentation of data, it is preferable to write Eqs. 56 and 57 in dimensionless terms as follows:

$$\frac{\sigma_y^2}{h_i^2} = 2\ \alpha_2 (x_i/h_i) \tag{58}$$

$$\frac{\sigma_y^2}{b_i^2} = 2\ \beta_e (x_i/b_i) \tag{59}$$

According to Eqs. 58 and 59, a plot of σ_y^2/h_i^2 versus x_i/h_i, or a plot of σ_y^2/b_i^2 versus x_i/b_i would show a straight line relationship depending on whether depth or width is the most influential channel parameter. Then, α_2 or β_e would be equal to one-half of the slope of the straight line of the appropriate plot.

3.4 Diffusion Factor

3.4.1 Variance Method:

Following Eq. 51b, a graph of σ_q^2 versus x should give a straight line of slope ($2\ D_y$), thus enabling a determination of the diffusion factor, D_y. From Eqs. 14b and 51b, we can get the following dimensionless relationship:

$$\frac{\sigma_q^2}{Q^2} = 2\ \beta(x/b) \tag{60}$$

Thus, we can estimate the nondimensional coefficient, β, from a plot of σ_q^2/Q^2 versus (x/b). When the channel widths vary with x in a given stream stretch, (x/b) is replaced by (x_i/b_i) as outlined previously.

3.4.2 Simplified Method:

A simple method for the determination of β can be developed from the following equation for the peak concentration of a conservative tracer, obtained from Eq. 19 with n = 0:

$$c(\ x,\ 0) = c_{max} = c_a(\ \pi\ \phi\ )^{-1/2}, \tag{61}$$

in which c_{max} = maximum concentration at a distance x below the outfall and $c_a = c_e Q_e/Q$. Substituting $\phi = \beta x/b$ from Eq. 15 and simplifying, we get:

$$(c_a/c_{max})^2 = (\ \pi\ \beta)(x/b) \tag{62}$$

Thus, a plot of $(c_a/c_{max})^2$ versus (x/b) would result in a straight line of slope, $s = (\pi\beta)$; then $\beta = s/\pi$.

This method is much simpler than the variance method, requiring channel widths and peak concentrations at various transects below the outfall, and values of c_e, Q_e and Q.

3.5 Computer Program for Data Analysis

As stated previously the computer program MIXANDAT (MIXing zone - ANalysis of DATa) is used to analyse the data collected during field surveys. The program is set up to accept data at up to eight transects and up to 60 values of lateral distances, depths, velocities and concentrations at each transect, as well as up to five water quality constituents at each transect. A table of widths, depths, velocities, concentrations and mass flux values is printed out at each transect for each of the constituents. The concentration ratios, (c/c_a) and (c/c_{atr}) are also included in the printout, where c_{atr} is the average concentration at a transect obtained from total net flux at the transect divided by the river discharge. The latter ratio would account for any differences in mass flux values of conservative tracers at various transects and thus it is better suited for comparison with predictions during model validation studies.

The variance values at each transect, computed from different methods by considering the distribution of each water quality constituent, are also tabulated, along with dimensionless variance values suitable for plotting. A list of the program is provided in Appendix B together with a list of the variable names. (Typical outputs of the program representing the results of the case study described in Chapter IV, are included in Appendix C.)

IV CASE STUDY I - GRAND RIVER BELOW WATERLOO

4.1 Field Studies

4.1.1 General Description:

The effluent from the Waterloo WPCP is discharged into the Grand River just below the Bridgeport bridge. At the time of these studies, the wastewater was being treated in a conventional activated sludge plant of 0.32 m^3/sec (6 MGD) design capacity. (Since then, the hydraulic capacity of the plant has been doubled). The treated effluent is discharged through a 61 cm pipe outlet, located near the left bank (looking upstream); the outfall pipe protrudes about 3.5 m into the river. A view of the river in the vicinity of the outfall is shown in Fig. 7(a).

Transverse dispersion studies were carried out in a 3.4 km stretch of the river below the outfall on June 12, 1975, and August 12, 1975. The general layout of the test section is shown in Fig. 7(b). Cross-sectional distributions of the various parameters of interest were monitored at five transects. The transects are identified on Fig. 7(b) by A1, B1, C1, D1 and E1 for the June test, and by A2, B2, C2, D2 and E2 for the August test[2]. The locations of first, second and fifth transects were different during the two tests. The distance below the outfall to each transect and the values of top width, mean depth and mean velocity at each transect are summarized in Table 2 (see Section 4.2). For general explanation purposes, the transects are referred to as A, B, C, D and E, except where distinction is warranted.

The channel alignment is fairly straight between the outfall and Transect C. There is a sharp bend between C and D, followed by a mild bend between D and E. The channel is very wide at Transect B.

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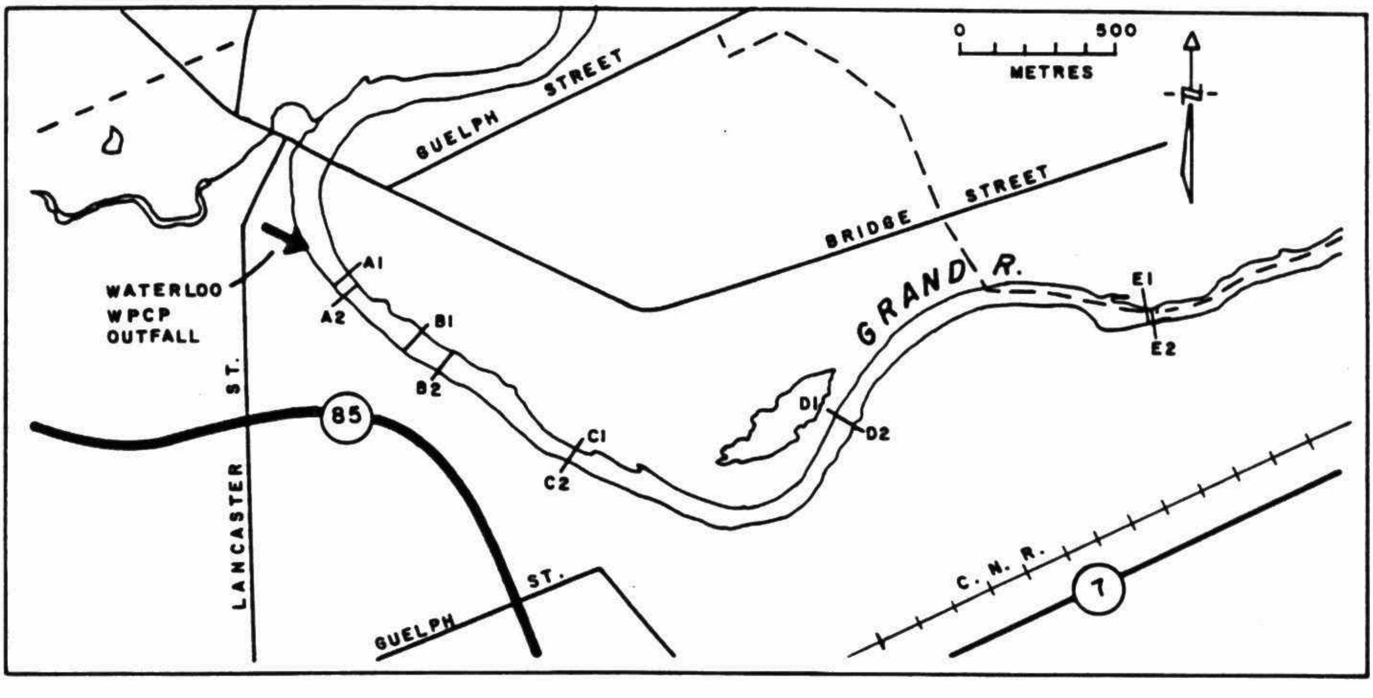
[2] In a previous publication (Ref. 9), some of the distances and concentrations were incorrectly stated due to an oversight in data extraction. The present report includes corrected data.

(a) A View of the Waterloo WPCP Outfall



(b) SCHEMATIC LAYOUT

FIG. 7. VIEW OF OUTFALL AND LAYOUT OF STUDY STRETCH - GRAND RIVER

The stream bed consists primarily of sand and gravel with some cobbles and large rocks. A few riffles are present in the reach AB; and there is a small island just upstream of Transect E.

During the June test, the streamflow rate was 12.15 m^3/sec immediately above the outfall and the effluent flow rate was 0.39 m^3/sec; during the August test, the corresponding flow rates were 9.76 m^3/sec and 0.34 m^3/sec. The streamflow rates at successive transects were prorated to account for potential accrual of discharge through seepage; the prorating was based on equal yield per unit area. The prorated discharge values at various transects, presented in Table 2 (see Section 4.2), are seen to vary from 12.54 to 12.66 m^3/s during the June test and from 10.10 to 10.14 m^3/s during the August test. The cross-sectional depth profiles were obtained at all transects, whereas velocity distributions were measured at Transects B and D only during each test; the depth profiles are showin in Fig. 8. The top widths and average depths and velocities in the study stretch vary in the range 36 to 85 m, 0.45 to 0.7 m and 0.2 to 0.55 m/sec, respectively (see Table 2 in Section 4.2).

4.1.2 Tracer Sources:

A solution of Rhodamine WT dye was prepared by mixing a known volume of 20% stock solution of the dye with river water in an 18-litre capacity constant head tank. The dye solution was injected through a plastic tube into the outfall pipe near the effluent outlet. The volumetric release rates of dye solution were 100 and 80 ml per minute during the June and August tests, respectively; the release rates were maintained for a duration of 1.5 to 2 hours. During the June survey, the concentration of dye injected from the constant head tank was measured to be 42.81 gm per litre; whereas during the August survey the concentration was estimated to be 25 gm per litre. Table 1 shows the concentration and mass flux rates of dye in the effluent.

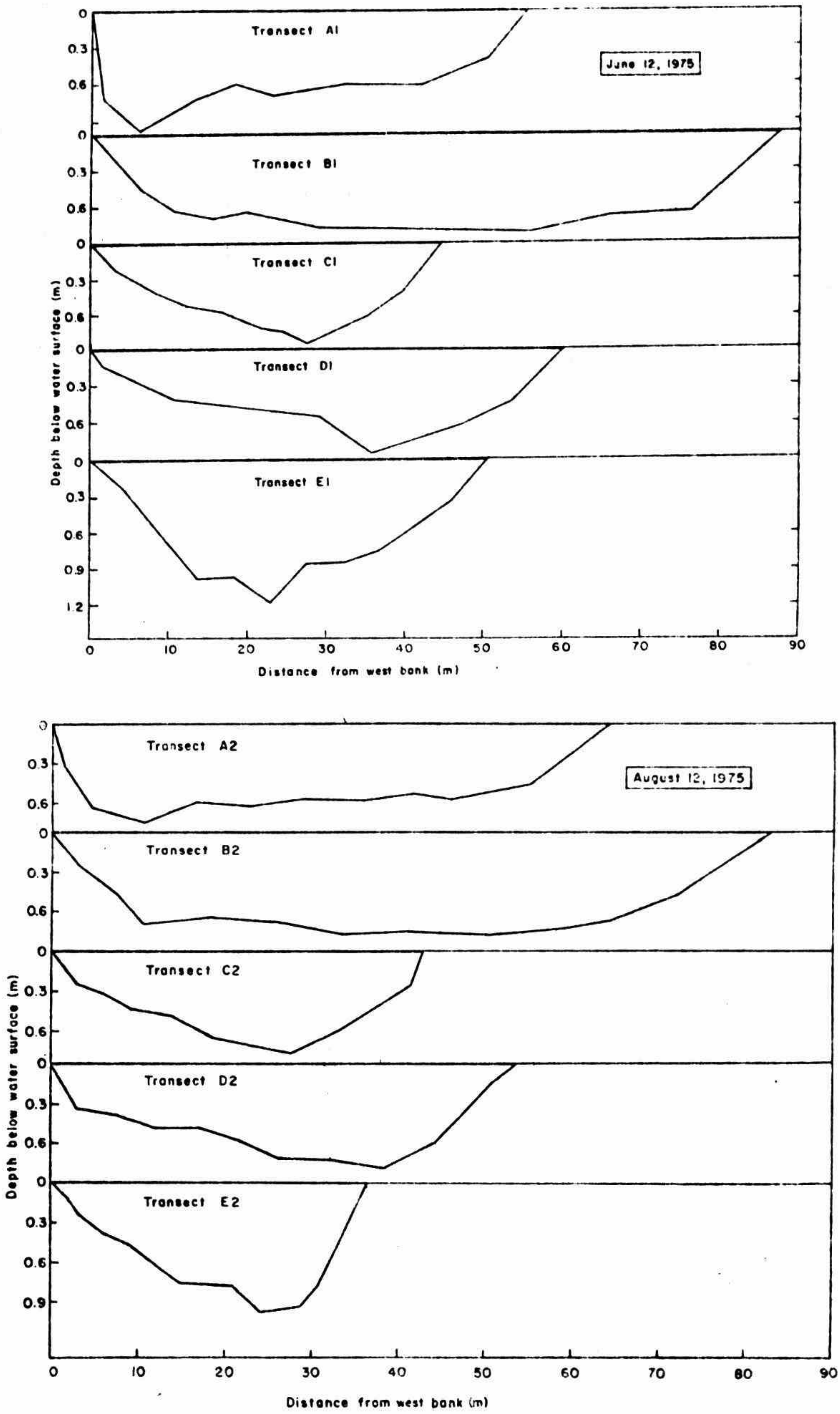


FIGURE 8. DEPTH PROFILES - GRAND RIVER

The materials normally present in water and considered in this study as conservative tracers include chloride ion and dissolved solids. Generally their concentrations in municipal wastewater effluents are higher than the background levels in the river. The concentration of dissolved solids can be determined by measuring specific conductance (or conductivity). In Ontario rivers and lakes, the concentration of dissolved solids (mg/L) is approximately 0.65 times specific conductance (umho/cm) when the latter is less than 400 umhos/cm (18). The conductance levels in the effluent and in several river water samples were higher than 400 umhos/cm. It is assumed, however, that these higher conductance values are also directly proportional to the concentrations of dissolved solids. This assumption seems reasonable in view of the fact that the dissolved solids concentration in a conventional activated sludge plant effluent is usually quite high. For the sake of simplicity, conductivity values are used in various analyses instead of dissolved solids concentrations.

TABLE 1: EFFLUENT TRACER DATA - GRAND RIVER

Test Date	June 12, 1975		August 12, 1975	
Tracer	Concentration	Mass Flux	Concentration	Mass Flux
Rhodamine dye	183 ug/L	71.4 mg/s	98 ug/L	33.3 mg/s
Chloride ion	270 mg/L	105.5 gm/s	358 mg/L	117.2 gm/s
Conductivity[1]	1571 umho/cm	613.0 units	2257 umho/cm	767.0 units

[1] Note: 1 umho/cm = 0.65 mg/L dissolved solids (approx.)
1 unit = 1 (umho-m^3)/(cm-sec)

The background concentrations in river water were as follows: chloride ion = 12 mg/L during both tests; conductivity = 390 and 350 umhos/cm during the June and August tests, respectively. The measured effluent concentrations of chloride ion and conductivity were found to result in too low values of effluent mass flux of

these parameters in comparison to the corresponding mass flux values at various transects. Hence, the concentrations were calculated as follows: The mass flux of each tracer at Transect A was computed by using the measured cross-sectional distributions (as outlined later in Section 4.2) and the corresponding background flux was subtracted; the net value was assumed to be equal to the effluent mass flux. The effluent concentration of each tracer was then computed by dividing the net flux value by the effluent discharge. The computed effluent concentrations and mass flux rates of chloride ion and conductivity are given in Table 1.

4.1.3 Concentration Measurements:

During each survey, data were gathered on the lateral distributions of dye, chloride ion and conductivity (except for the dye distribution at B1 during the June survey). During the cross-sectional depth and velocity measurements, water samples were taken for determination of chloride ion concentrations, and specific conductance measurements were taken _in situ_ with a conductivity meter; the samples were then analysed in the laboratory for chloride ion concentrations (18). The distribution of dye was measured with a Turner Designs Model 10-05 Fluorometer equipped with continuous flow attachment, and was directly recorded on a strip chart. The equipment was set-up in a 5 m (16 ft.) canoe, a view of which is shown in Fig. 9. At each transect, at least two tracings of the dye distributions were obtained at intervals of about 5 to 10 minutes to make sure that the dye concentration profile had attained a steady state. (The dye distribution at Transect B1 was not measured since the marker float installed at the transect was missing). The depth of water near the channel banks at some of the transects was too small to permit free passage of the canoe, thus making it extremely difficult to obtain the dye tracings in a width of 2 to 4 meters near each shore; in such cases, the concentrations were obtained by extrapolation. At some of the transects, dye tracing was carried out about 1 to 2 hours after the completion of sampling for chloride ion and conductivity; and hence the same parcel of water was not necessarily followed in the chloride ion and conductance sampling.

FIG. 9. VIEW OF EQUIPMENT SET-UP IN CANOE

The measured transverse distributions of dye, chloride ion and conductivity at various transects below the outfall are shown in Fig. 10 for the June and August tests. The plots indicate that the peak concentrations of chloride ion and conductivity occur at or near the outfall shoreline of the river channel at all transects, whereas the positions of the peaks of the dye distributions occur at some point away from that shoreline of the channel. The latter observation may be due to the presence of stagnant areas near the shore zones (see flow distribution curves in Fig. 11). The occurrence of the peak concentration of chloride ion near the channel shore may have been due to the input of chlorides through ground water seepage or to the presence of stagnant areas near the shore. The occurrence of the conductivity peak near the shoreline can be attributed to these two factors, as well as due to biochemical processes near the shore where organic deposits and decaying vegetation are usually present. The mass flux values of chloride ion and conductivity (presented in Fig. 12, Section 4.2) were seen to fluctuate at various transects, indicating the possible presence of such additional sources and sinks of these tracers. The fact that samples for chloride ion and conductivity were not necessarily collected by following the same plug of water may also have had an effect on the concentrations measured at successive transects. Further investigations are desirable to determine the reasons for the observed differences in the locations of the peaks of dye, chloride and conductivity. However, such studies are beyond the scope of this paper.

A comparison of the tracer distributions at Transect B with those at other transects indicates a greater spread of each tracer at this transect. For example, during the June test, the spread of tracers was about 38 m at Transect B1 where the channel width was 86.87m; whereas the spread was about 27.4 m at Transect C1, the channel width being only 50.3 m. These results indicate that the transverse spreads at successive transects would increase or decrease in some proportion to the channel widths.

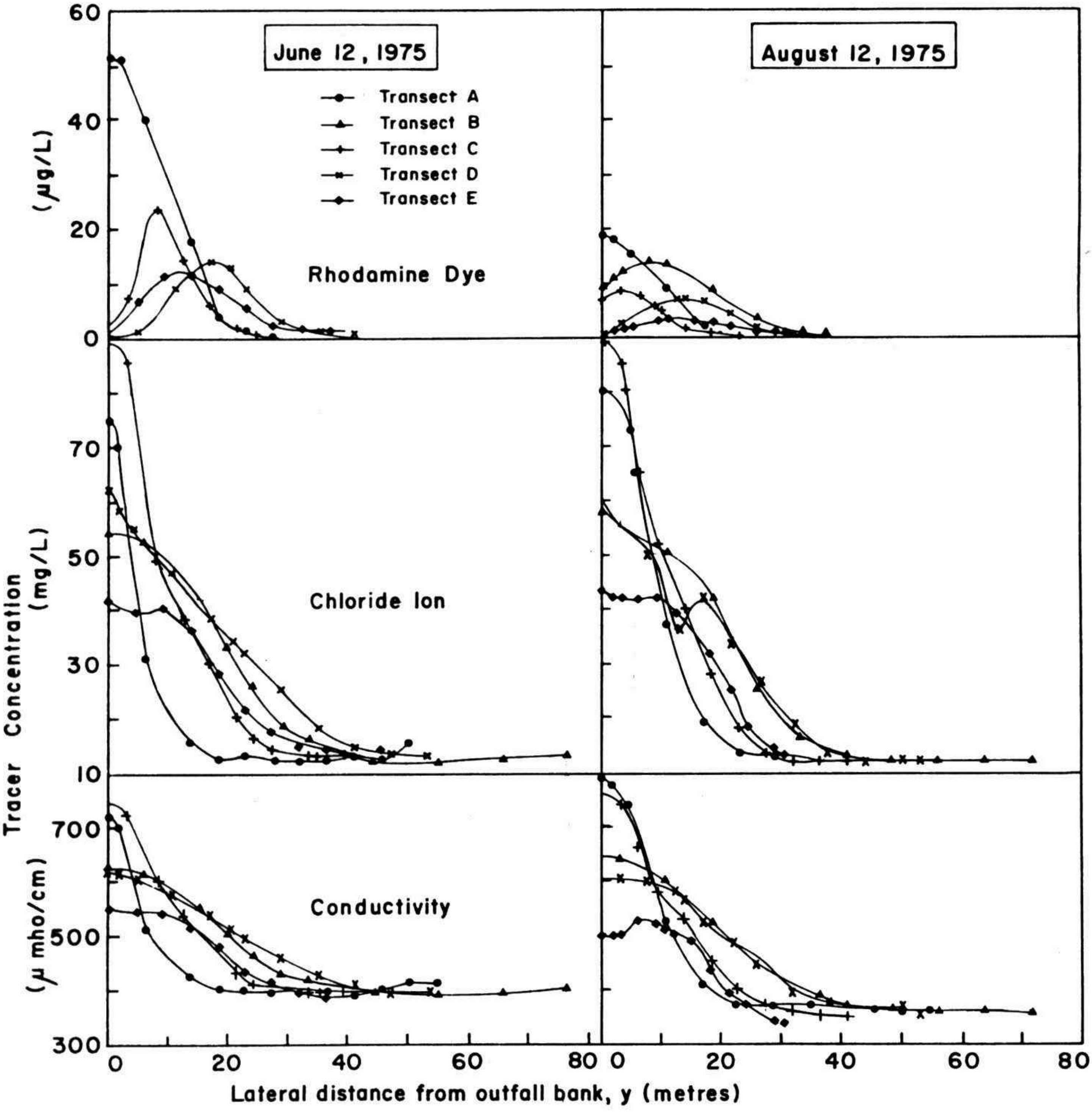


FIGURE 10. PROFILES OF DYE, CHLORIDE ION AND CONDUCTIVITY

4.2 Analysis of Data

The computer program MIXANDAT was used to calculate cross-sectional distribution of flows, average values of depth and velocity, as well as mass flux and variance at each transect by using the depths and concentration values of dye, chloride and conductivity. All computations are based on trapezoidal approximations, as stated in Chapter III. The computational results of MIXANDAT program are presented in Appendix C. The following subsections describe the details of various computations.

4.2.1 Flow Distribution Curves:

The cross-sectional distributions of streamflow at various transects, obtained as outlined in subsection 3.2.2 (Chapter III), are shown in the tables in Appendix C. For plotting purposes, the distributions were normalized with respect to channel width and total streamflow; these values are also tabulated in Appendix C. Nondimensional plots q/Q versus y/b are presented in Fig. 11 for the June and August tests. Note that the distributions at Transects B and D are obtained from the measured velocity profiles whereas the other three distributions are derived from synthetic velocity profiles using the Manning's equation.

From the plots presented in Figure 11, it is possible to identify dead zones. For example, the June test results indicate that about 10% of the width near the outfall bank was stagnant. The August test results indicate the presence of a stagnant zone at Transect E2 between y/b = 0.6 to 0.8. The plots can also be used to determine the relative magnitudes of discharge within a given fraction of channel width. As an example, the June test results show that the partial cumulative discharge within 50% of the channel width (i.e., y/b=0.5) was the smallest at Transect D1 (q/Q = 0.24) and the largest at Transect A1 (q/Q = 0.64).

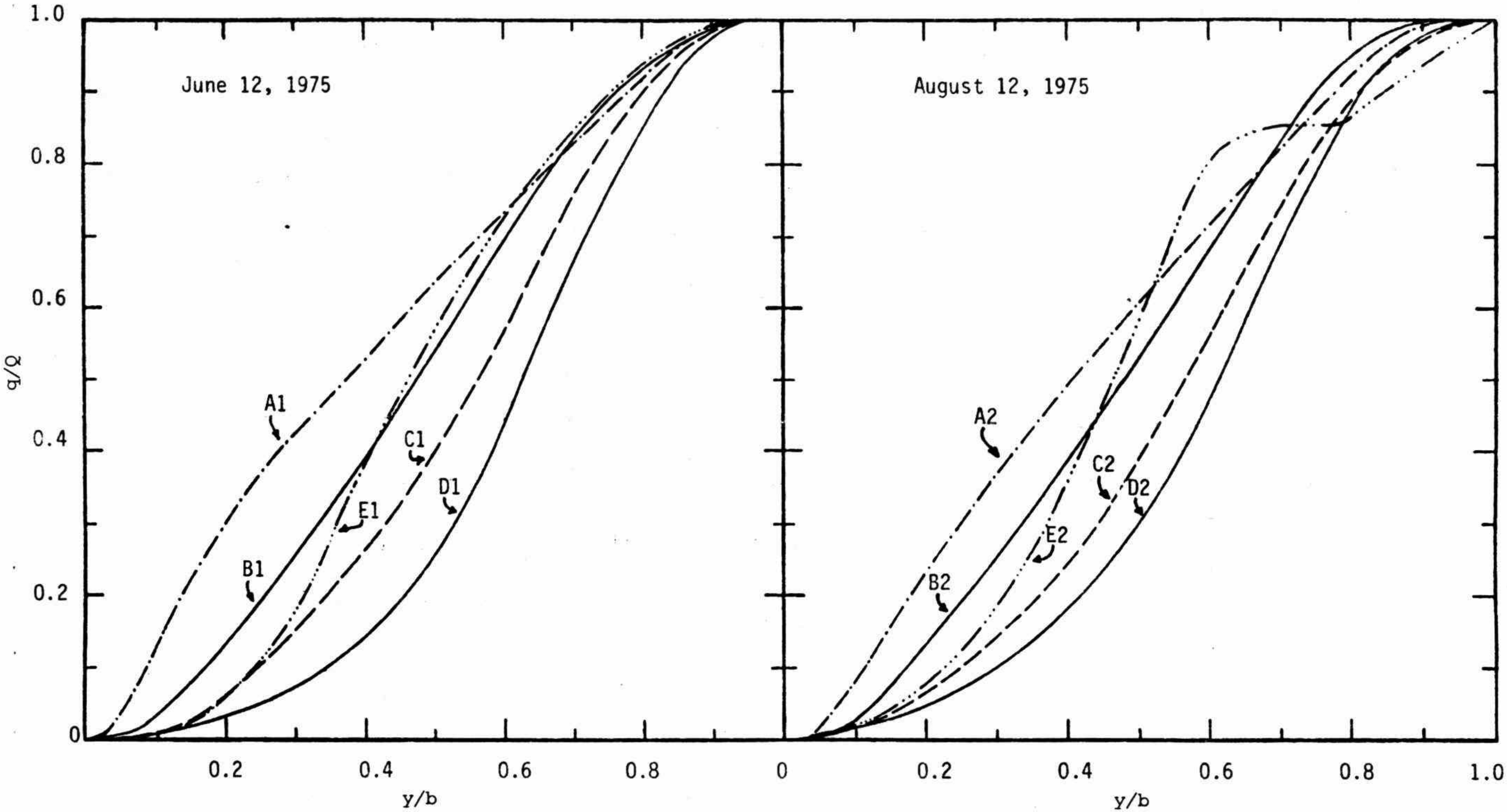


FIG. 11. FLOW DISTRIBUTION CURVES

4.2.2 Tracer Mass Flux:

The mass flux of each tracer at a given transect was determined from Eqs. 49 and 50 (see subsection 3.2.4). Plots of mass flux of each tracer versus distance below the source are shown in Fig. 12. The June test results show that the mass flux value of dye at Transect A1 is higher than that discharged at the outfall, possibly due to the effect of the stagnation of concentrated dye near the west shore immediately below the outfall. The flux values of chloride ion and conductivity at various transects also differ from the corresponding effluent flux values given in Table 1.

(Note: An examination of the measured concentration distributions of chloride ion and conductivity indicated that the concentration values outside the effluent plume differed slightly from the corresponding background values in the river just upstream of the outfall. Therefore, the background values at each transect were estimated by examining the measured distributions. The background values of chloride ion were 12-13 mg/L for both tests, whereas the conductance values were 390-395 umhos/cm and 340-360 umhos/cm for the June and August tests, respectively, as shown in the tables in Appendix C.)

The estimated mass flux of tracer at each transect is indicative of the recovery of the tracer discharged in the effluent. A significant change in the tracer flux at a transect in relation to the input rate tends to indicate that either the system had not attained steady state conditions during sampling, or there were other sources and sinks of the tracer. The general decrease in the mass flux of dye in the longitudinal direction may be attributed to the loss of dye trapped in stagnant pools. The observed deviations in the mass flux of chlorides and conductivity at various transects may be due to the reasons described in the previous section.

4.2.3 Shape-Velocity Factor:

The values of the shape-velocity factor, ψ, at various transects computed by Eq. 48 are given in tabulated output of MIXANDAT program

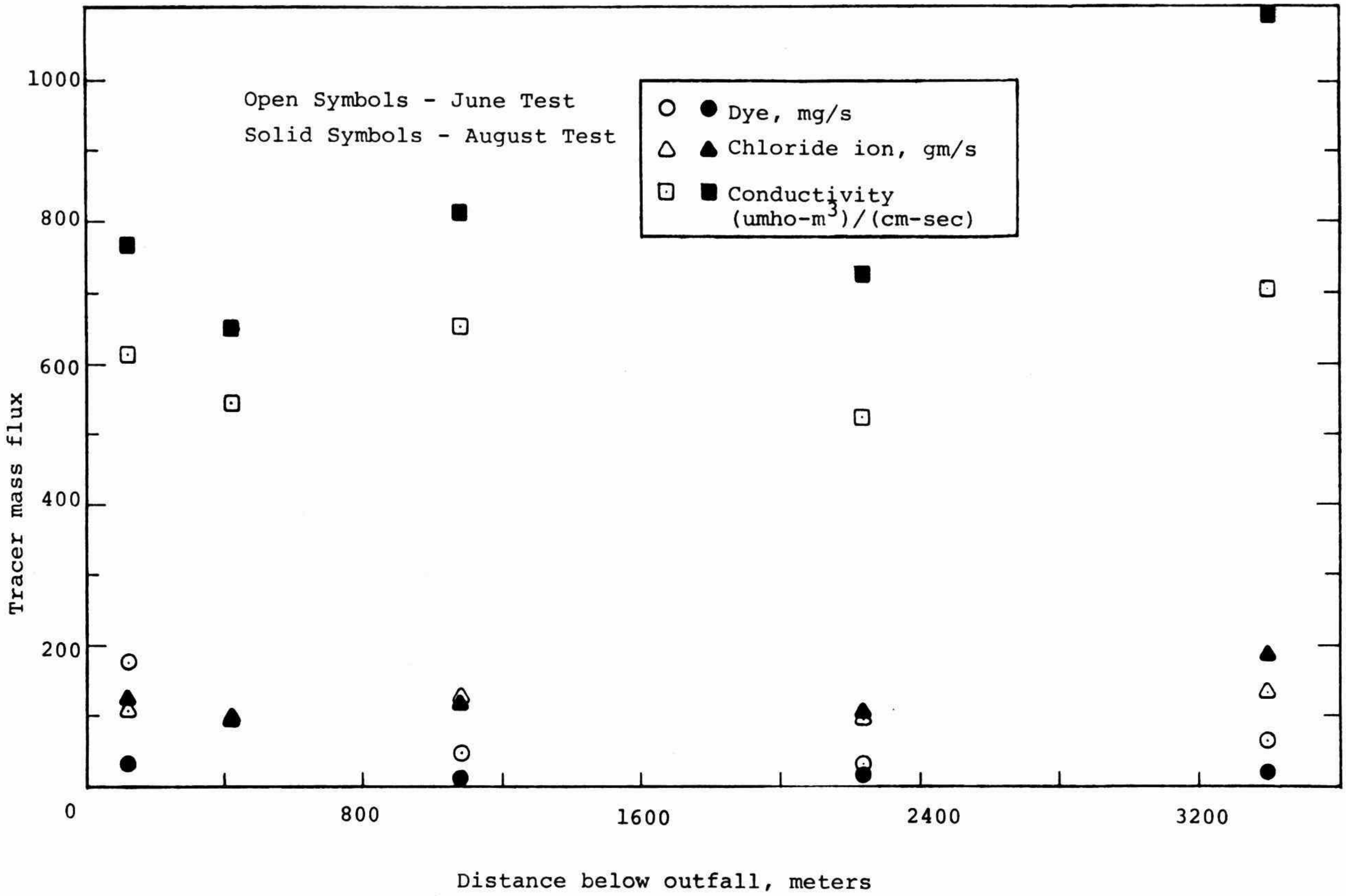


FIGURE 12. TRACER MASS FLUX

in Appendix C and are summarized in Table 2. The values at various transects are in the range 1.35 to 2.53 and are within the range 1.0 to 3.2 reported by Beltaos (2).

4.2.4 Transverse Dispersion Coefficient:

In order to evaluate the transverse dispersion coefficient, e_y, the stretch of the river is considered to be a channel of straight alignment, although the lower portion of the study stretch meanders. This assumption seems reasonable in light of a review of transverse dispersion studies in meandering flumes and rivers which indicates that in general, the presence of curvature of river bends would not be a dominant influence on the transverse dispersion characteristics (1).

The variance values of each tracer concentration and unit flux distribution curves were determined by taking moments with respect to the outfall shore of the stream channel; the variance values were also computed from the peak concentrations of each tracer at various transects. The values obtained by the three methods are presented in the tables in Appendix C and are summarized in Table 2. The relatively large spread of the tracers observed at Transect B is reflected in higher values of variance. In contrast, the variance values at Transect C are lower than those at B, in line with the smaller spread observed. At Transect D, which is located below a sharp bend, the variance values are considerably higher, indicating the possible effects of transverse velocities around the bend. Variance values of the unit flux distributions, σ_f^2, are generally higher than those obtained from the concentration distributions, whereas those of the peak concentration method have the lowest values. The nondimensional variance values σ_y^2/h^2 and σ_y^2/b^2 are also given in the tables in Appendix C.

Plots of variance versus distance are presented in Fig. 13 to examine the applicability of Eq. 51a to determine e_y. These plots show that the longitudinal variation of the variance of each tracer distribution generally follows the same pattern, irrespective of the

TABLE 2. SUMMARY OF HYDRAULIC PARAMETERS AND VARIANCE VALUES - GRAND RIVER

Test Date	June 12, 1975					August 12, 1975				
Transect	A1	B1	C1	D1	E1	A2	B2	C2	D2	E2
Geometric and Hydraulic Data										
Distance, x (m)	122.0	427.0	1082.0	2240.0	3398.0	183.0	533.0	1082.0	2240.0	3429.0
Discharge, Q (m^3/s)	12.54	12.58	12.60	12.63	12.66	10.10	10.11	10.11	10.12	10.14
Width, b (m)	54.86	86.87	44.2	59.44	50.29	64.01	82.3	42.67	53.34	36.58
Depth, h (m)	0.62	0.62	0.52	0.49	0.69	0.54	0.60	0.50	0.51	0.45
Velocity, u (m/s)	0.37	0.23	0.55	0.43	0.36	0.29	0.20	0.47	0.37	0.62
Shape-Velocity Factor	1.41	1.46	1.79	2.19	1.97	1.35	1.54	1.73	2.00	2.53
Variance Values (m^2)										
Rhodamine WT Dye:										
Method 1*	18.94	-	15.79	42.09	56.05	15.10	63.31	16.52	53.49	47.85
Method 2*	75.88	-	117.01	385.46	249.95	62.93	224.92	57.42	255.26	274.52
Method 3*	75.15	-	165.68	587.48	320.58	81.93	330.15	107.66	399.65	253.49
Chloride Ion:										
Method 1	5.16	66.18	16.89	55.01	60.09	13.88	71.04	14.60	57.25	58.77
Method 2	35.08	234.96	96.28	273.41	187.56	72.40	250.08	102.17	249.00	158.48
Method 3	41.79	364.39	195.33	629.49	295.97	93.26	377.75	214.35	497.29	213.27
Conductivity:										
Method 1	6.40	61.87	20.32	73.42	54.90	17.31	73.57	24.28	88.92	50.14
Method 2	48.93	256.84	99.84	286.50	171.00	107.46	340.79	130.50	297.44	149.43
Method 3	53.47	407.96	193.72	648.36	281.73	127.22	519.41	252.88	572.13	202.97

* Method 1: Maximum concentration method
Method 2: Moment method - concentration distribution
Method 3: Moment method - unit flux distribution

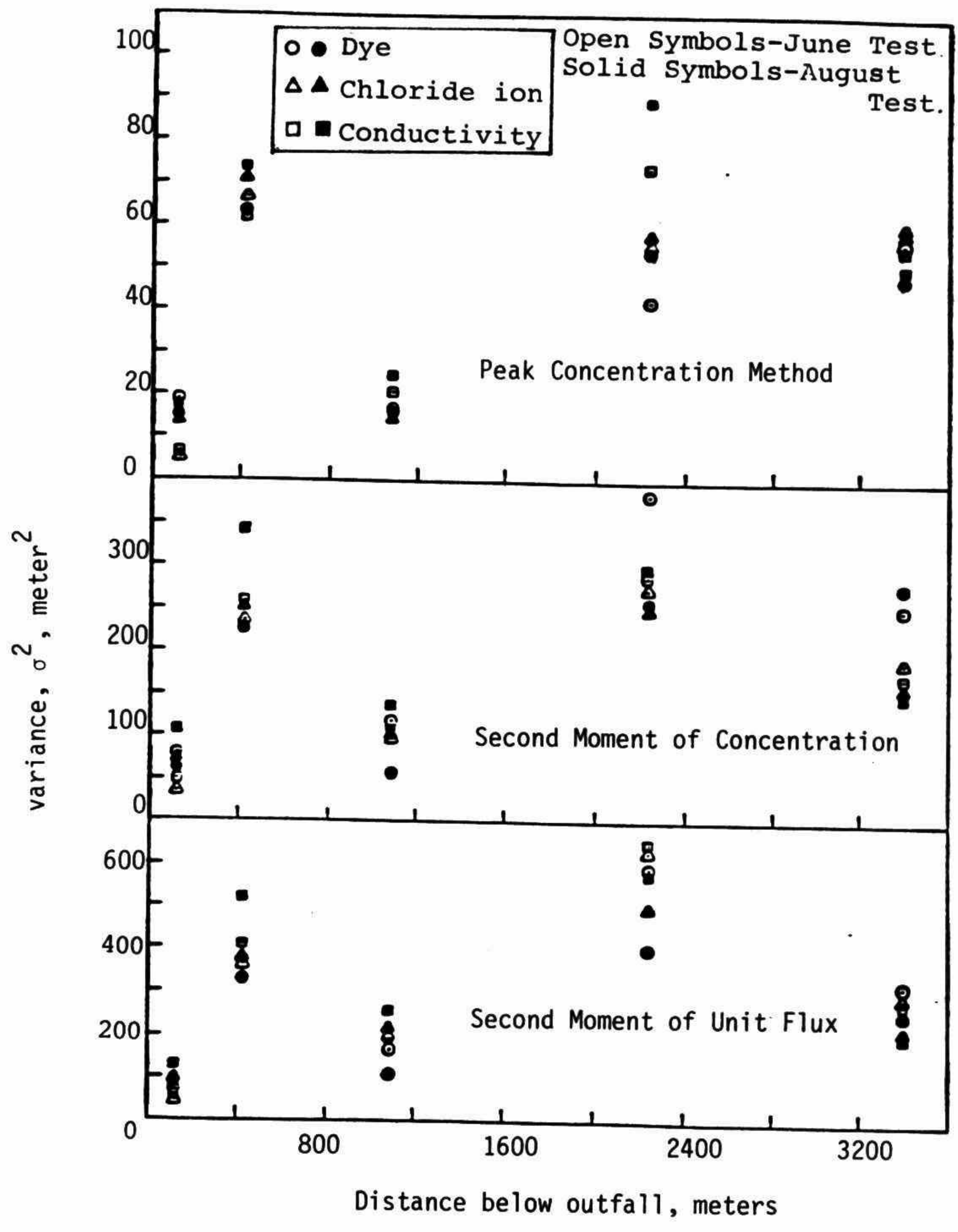


FIGURE 13. VARIANCE VERSUS DISTANCE PLOTS

method used to compute the variance values. However, a steady growth of variance at successive transects is not displayed in these plots, indicating a lack of fit of data to Eq. 51a.

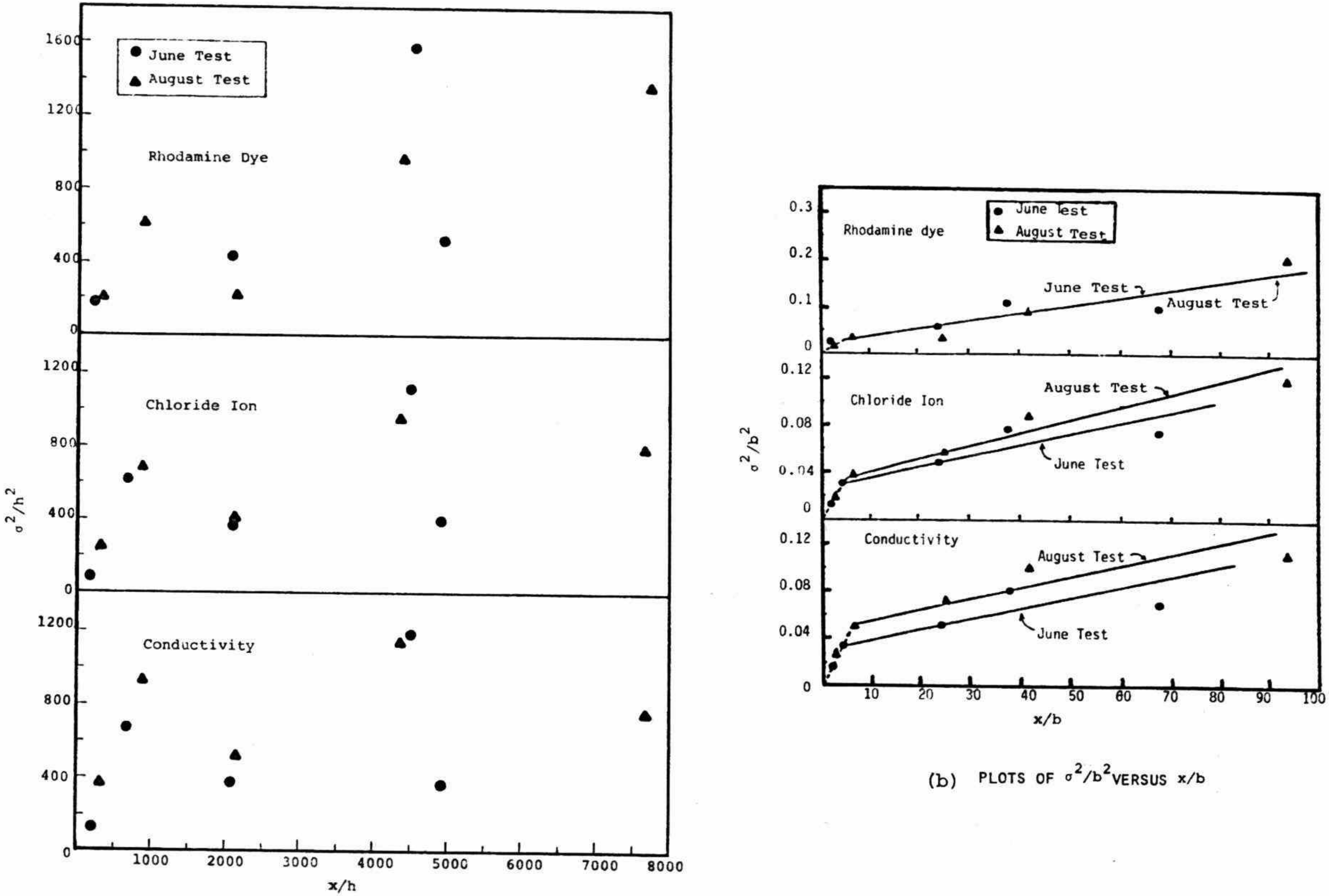
In order to examine the applicability of Eq. 58, plots of σ_y^2/h^2 versus x/h, shown in Fig.14(a) were prepared by using the variance values determined from the second moment of concentration distribution curves. An examination of the plots shows a large scatter of the data points, suggesting a limitation of the applicability of Eq. 58; hence, no attempt was made to determine the values of α_2 from these plots.

Figure 14(b) shows plots of σ_y^2/b^2 versus x/b for the June and August tests; variance values given by the second moment of concentration curves are used here also. The scatter of the data points in Fig. 14(b) is much smaller than that in Fig. 14(a), indicating a better fit of the data to Eq. 59. Thus, the results of the field tests tend to support the hypothesis that the transverse dispersion coefficient is a function of width (rather than depth) and velocity of flow in a shallow river.

In Fig. 14(b), the rate of growth of variance in the region immediately below the outfall is seen to be higher than that at farther distances. This diffusion pattern is in agreement with the results of a previous field study by Holley and Abraham (12), and is attributed to initial mixing mechanisms below a bank outfall. The dispersion characteristics are indicated by the growth rates of variance at large distances. For this reason, straight lines of best fit, drawn by neglecting the influence of data points which are very close to the outfall, will be considered to determine the dispersion coefficients.

From the slopes of the straight lines of the various plots shown in Fig. 14(b), the following values of β_e are obtained:

Tracer	June Test	August Test
Rhodamine dye	8.3×10^{-4}	8.3×10^{-4}
Chloride ion	4.5×10^{-4}	5.5×10^{-4}
Conductivity	4.5×10^{-4}	4.7×10^{-4}



(a) PLOTS OF σ^2/h^2 VERSUS x/h

(b) PLOTS OF σ^2/b^2 VERSUS x/b

FIG. 14. DIMENSIONLESS VARIANCE vs. DISTANCE PLOTS

These values are in the general range of the results of laboratory studies presented by Lau and Krishnappan (16). The results of the dye tracer plots are seen to be higher than those of the other two tracers. For practical purposes, it appears reasonable to use either chloride ion or conductivity as a tracer in effluent dispersion studies in streams and rivers, although some limitations are evident as stated earlier (Section 4.1).

Table 3 shows values of e_y at each transect, determined from the hydraulic data presented in Table 2 and the nondimensional coefficients given above. The values of e_y lie in the range 0.0077-0.02 m^2/s, and are within the range observed in other shallow streams (1,29). A comparison of the e_y values of the two tests indicates that, in general, values at corresponding transects are almost the same. Thus, the observed change in streamflow rates did not significantly affect e_y in the study stretch of the Grand River. It is also seen that e_y is generally smaller at a wider transect, in agreement with the findings of Lau and Krishnappan (16).

TABLE 3. VALUES OF TRANSVERSE DISPERSION COEFFICIENT, e_y

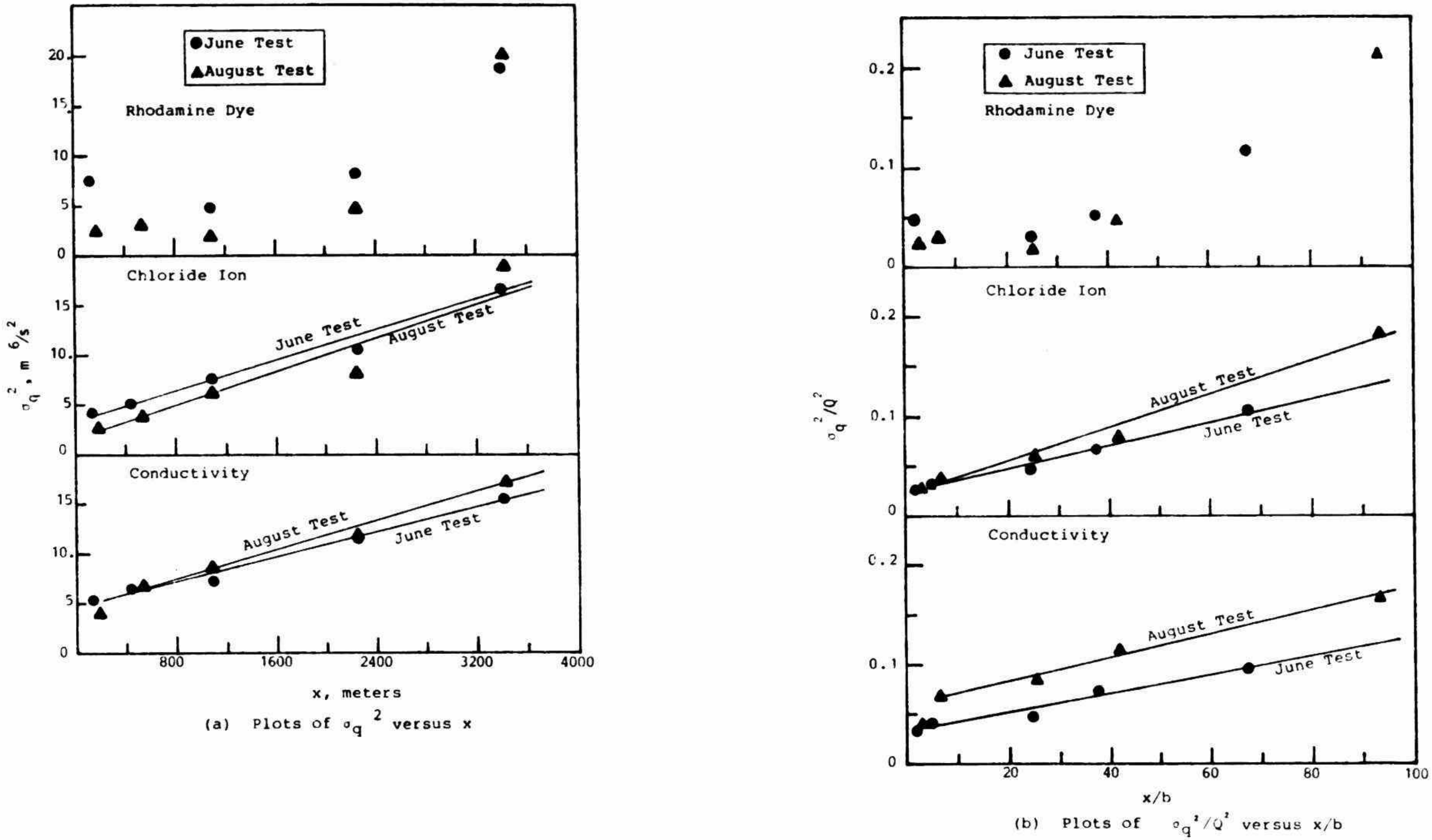
Date of Test	Transect	e_y in m^2/s		
		Rhodamine dye	Chloride ion	Conductivity
June 12, 1975	A1	1.69×10^{-2}	9.14×10^{-3}	9.14×10^{-3}
	B1	1.66×10^{-2}	8.99×10^{-3}	8.99×10^{-3}
	C1	2.02×10^{-2}	10.90×10^{-3}	10.90×10^{-3}
	D1	2.12×10^{-2}	11.50×10^{-3}	11.50×10^{-3}
	E1	1.54×10^{-2}	8.37×10^{-3}	8.37×10^{-3}
August 12, 1975	A2	1.54×10^{-2}	10.20×10^{-3}	8.72×10^{-3}
	B2	1.36×10^{-2}	9.05×10^{-3}	7.74×10^{-3}
	C2	1.66×10^{-2}	11.00×10^{-3}	9.40×10^{-3}
	D2	1.64×10^{-2}	10.90×10^{-3}	9.28×10^{-3}
	E2	1.88×10^{-2}	12.50×10^{-3}	10.62×10^{-3}

4.2.5 Diffusion Factor:

The variance values, σ_q^2, determined from the second moment of the c-q distributions of each tracer are tabulated in Appendix C, and are summarized in Table 4. (Note: The values of σ_q^2 obtained from the peak concentration method are also given in the tables in Appendix C; but they were not used in the analysis.) Plots of σ_q^2 versus x, presented in Fig. 15(a), indicate that the data points of chloride ion and conductivity generally follow Eq. 51b, whereas the dye tracer data points show significant deviations; therefore, the values of D_y were not estimated from the latter plots. From the chloride ion and conductivity data plots, D_y values are found to vary from 0.0014 to 0.0019 m^5/s^2, the August test values being slightly higher than those of the June test.

Following Eq. 60, Fig. 15(b) shows the dimensionless plots, σ_q^2/Q^2 versus x_i/b_i. As seen in Fig. 15(a), the data points of the chloride ion and conductivity tracers indicate a linear relationship whereas those of the dye tracer show some deviations. The values of β determined from the slopes of the straight lines of chloride ion and conductivity plots are summarized in Table 4; the August test results indicate slightly higher values of β than those of the June test.

The values of β given in Table 4 represent average values for the study stretch of the river and can be used as inputs during model calibration studies. However, it may be necessary to adjust the values during the calibration-verification studies in order to obtain a satisfactory agreement between the observed and predicted concentration distributions; in particular, it may be necessary to use a different value of β for each transect. The resulting values, β_i, are then used in further modelling studies. In this context, the values presented in Table 4 could represent initial estimates; nevertheless, these values are of primary importance in model development and application.



(a) Plots of σ_q^2 versus x

(b) Plots of σ_q^2/Q^2 versus x/b

FIGURE 15. DIFFUSION FACTOR - GRAND RIVER

TABLE 4. SUMMARY OF VARIANCE VALUES AND DIFFUSION FACTORS.

Test Date	Tracer	σ_q^2 at stated transect, m^6/s^2 A	B	C	D	E	D_y m^5/s^2	β
June 12, 1975	Rhodamine dye	7.58	-	4.91	8.18	18.72	-	-
	Chloride Ion	4.06	5.19	7.48	10.51	16.84	0.0017	5.66×10^{-4}
	Conductivity	5.22	6.44	7.28	11.51	15.43	0.0014	5.08×10^{-4}
August 12, 1975	Rhodamine dye	2.44	3.09	1.66	4.18	22.06	-	-
	Chloride Ion	2.89	3.87	6.12	8.18	18.76	0.0019	8.00×10^{-4}
	Conductivity	4.14	6.81	8.65	11.79	17.13	0.0017	6.67×10^{-4}

4.3 Model Validation Studies

The chloride ion and conductivity distributions shown in Fig. 10 exhibit the characteristics of a shore-attached effluent plume. Since these distributions show similar patterns, only the chloride ion distribution data were used to calibrate and verify the model. The dye distribution data has not been used in the modelling studies since those distributions show shifts in the positions of peak concentrations.

4.3.1 Calibration of Model:

The input data for calibration of the MIXCALBN model were obtained from the June test results given in Tables 1 to 3. However, the river discharge at each transect was taken as 12.54 m^3/s by neglecting the accrual of flow. (This assumption does not significantly affect the predictions, since the flow accrual in the study stretch is less than 10%.) Using the value of $\beta = 0.000566$ given in Table 4 as first estimate of β at each transect, concentration distributions of chloride ion at various transects were predicted (see Table 5); a comparison of these predictions with the observed distributions showed significant differences at all transects (see Fig. 16). Several runs were then carried out by

using different values of β_i for each of the five transects. The predictions of the run which were in close agreement with the observations, have been given in Table 5 along with the values of β_i (shown as BETA for each transect). Using these predictions, dimensionless plots of $c(x, q)/c_{av}$ versus q/Q, shown in Fig. 16, were prepared; the corresponding dimensionless data points obtained from the observed chloride ion distributions (presented in Appendix C) are also shown on the plots in Fig. 16 for comparison. (Note: For the reasons stated in Section 3.2, the observed data points plotted in Fig. 16 represent c/c_{atr} versus q/Q, where c_{atr} denotes the average concentration at a given transect.) From a comparison of the plots at Transects C1 and D1, some of the observed data near $q = 0$ are found to be higher than the predictions. This seems to be due to the presence of stagnation areas near the shoreline and is consistent with the cross-sectional distributions of flow shown on Fig. 11. The values of β_i obtained by this procedure range from 2.5×10^{-3} to 8.0×10^{-4} at various transects; they were accepted as satisfactory for further modelling studies. The difference between the values of β given in Table 4 and the values of β_i obtained from model calibration may be due to the approximations involved in the former procedure.

It is possible to make an approximate estimation of the metric coefficients from the general form of Eq. 14c (see Section 2.2), given by $\beta_i = (\bar{m}_x \psi \beta_e)_i$, where $\bar{m}_x$ represents the average value for a stretch. Substitution of the values of ψ given in Table 2 and $\beta_e = 4.5 \times 10^{-4}$ (from Table 3) in the above expression results in $\bar{m}_x = 3.94$, 3.04, 1.61, 0.81 and 1.12 for the transects A1 to E1, respectively. The first two values appear to be rather high, and may reflect the effects of protrusion of the outfall pipe on the diffusion pattern at the transects A1 and B1.

4.3.2 Model Verification Study:

The calibrated model was verified by comparing the observed and predicted chloride ion concentration distibutions for the August

TABLE 5. MODEL CALIBRATION RUNS- GRAND RIVER.

(a) Predictions with constant β_i

WATERLOO JUNE 12, 1975 * CHLORIDE DATA *

TRANSECT 1 BETA= 0.000566 RKS= 0.0

X	BW	HW	UW
122.000	54.860	0.620	0.370

QY	C(X,QY)	CUI	C/CA	QY/QT
0.0	133.5350	0.0	15.9025	0.0
1.25	18.3234	0.0	2.1821	0.1000
2.51	0.0473	0.0	0.0056	0.2000
3.76	0.0000	0.0	0.0000	0.3000
5.02	0.0000	0.0	0.0000	0.4000
6.27	0.0	0.0	0.0	0.5000
7.52	0.0	0.0	0.0	0.6000
8.78	0.0	0.0	0.0	0.7000
10.03	0.0	0.0	0.0	0.8000
11.29	0.0	0.0	0.0	0.9000
12.54	0.0	0.0	0.0	1.0000

TRANSECT 2 BETA= 0.000566 RKS= 0.0

X	BW	HW	UW
427.000	66.292	0.620	0.305

QY	C(X,QY)	CUI	C/CA	QY/QT
0.0	78.4629	0.0	9.3440	0.0
1.25	39.5232	0.0	4.7068	0.1000
2.51	5.0515	0.0	0.6016	0.2000
3.76	0.1638	0.0	0.0195	0.3000
5.02	0.0013	0.0	0.0002	0.4000
6.27	0.0000	0.0	0.0000	0.5000
7.52	0.0000	0.0	0.0000	0.6000
8.78	0.0000	0.0	0.0000	0.7000
10.03	0.0	0.0	0.0	0.8000
11.29	0.0	0.0	0.0	0.9000
12.54	0.0	0.0	0.0	1.0000

TRANSECT 3 BETA= 0.000566 RKS= 0.0

X	BW	HW	UW
1082.000	65.834	0.587	0.325

QY	C(X,QY)	CUI	C/CA	QY/QT
0.0	49.1199	0.0	5.8496	0.0
1.25	37.5441	0.0	4.4711	0.1000
2.51	16.7647	0.0	1.9965	0.2000
3.76	4.3734	0.0	0.5208	0.3000
5.02	0.6665	0.0	0.0794	0.4000
6.27	0.0593	0.0	0.0071	0.5000
7.52	0.0031	0.0	0.0004	0.6000
8.78	0.0001	0.0	0.0000	0.7000
10.03	0.0000	0.0	0.0000	0.8000
11.29	0.0000	0.0	0.0000	0.9000
12.54	0.0000	0.0	0.0000	1.0000

TRANSECT 4 BETA= 0.000566 RKS= 0.0

X	BW	HW	UW
2240.000	58.589	0.547	0.391

QY	C(X,QY)	CUI	C/CA	QY/QT
0.0	32.2056	0.0	3.8353	0.0
1.25	28.6918	0.0	3.4169	0.1000
2.51	20.2879	0.0	2.4160	0.2000
3.76	11.3859	0.0	1.3559	0.3000
5.02	5.0717	0.0	0.6040	0.4000
6.27	1.7930	0.0	0.2135	0.5000
7.52	0.5031	0.0	0.0599	0.6000
8.78	0.1121	0.0	0.0133	0.7000
10.03	0.0198	0.0	0.0024	0.8000
11.29	0.0028	0.0	0.0003	0.9000
12.54	0.0006	0.0	0.0001	1.0000

TRANSECT 5 BETA= 0.000566 RKS= 0.0

X	BW	HW	UW
3398.000	57.320	0.561	0.390

QY	C(X,QY)	CUI	C/CA	QY/QT
0.0	25.8636	0.0	3.0800	0.0
1.25	24.0065	0.0	2.8589	0.1000
2.51	19.1979	0.0	2.2862	0.2000
3.76	13.2270	0.0	1.5752	0.3000
5.02	7.8515	0.0	0.9350	0.4000
6.27	4.0153	0.0	0.4782	0.5000
7.52	1.7692	0.0	0.2107	0.6000
8.78	0.6717	0.0	0.0800	0.7000
10.03	0.2202	0.0	0.0262	0.8000
11.29	0.0650	0.0	0.0077	0.9000
12.54	0.0301	0.0	0.0036	1.0000

(b) Predictions with variable β_i

WATERLOO JUNE 12, 1975 * CHLORIDE DATA *

TRANSECT 1 BETA= 0.002500 RKS= 0.0

X	BW	HW	UW
122.000	54.860	0.620	0.370

QY	C(X,QY)	CUI	C/CA	QY/QT
0.0	63.5380	0.0	7.5666	0.0
1.25	40.5269	0.0	4.8263	0.1000
2.51	10.5165	0.0	1.2524	0.2000
3.76	1.1103	0.0	0.1322	0.3000
5.02	0.0477	0.0	0.0057	0.4000
6.27	0.0008	0.0	0.0001	0.5000
7.52	0.0000	0.0	0.0000	0.6000
8.78	0.0000	0.0	0.0000	0.7000
10.03	0.0000	0.0	0.0000	0.8000
11.29	0.0000	0.0	0.0000	0.9000
12.54	0.0	0.0	0.0	1.0000

TRANSECT 2 BETA= 0.002000 RKS= 0.0

X	BW	HW	UW
427.000	66.292	0.620	0.305

QY	C(X,QY)	CUI	C/CA	QY/QT
0.0	41.7405	0.0	4.9708	0.0
1.25	34.3777	0.0	4.0940	0.1000
2.51	19.2059	0.0	2.2872	0.2000
3.76	7.2783	0.0	0.8668	0.3000
5.02	1.8710	0.0	0.2228	0.4000
6.27	0.3262	0.0	0.0389	0.5000
7.52	0.0386	0.0	0.0046	0.6000
8.78	0.0031	0.0	0.0004	0.7000
10.03	0.0002	0.0	0.0000	0.8000
11.29	0.0000	0.0	0.0000	0.9000
12.54	0.0000	0.0	0.0000	1.0000

TRANSECT 3 BETA= 0.001300 RKS= 0.0

X	BW	HW	UW
1082.000	65.834	0.587	0.325

QY	C(X,QY)	CUI	C/CA	QY/QT
0.0	32.4111	0.0	3.8598	0.0
1.25	28.8322	0.0	3.4336	0.1000
2.51	20.2969	0.0	2.4171	0.2000
3.76	11.3070	0.0	1.3465	0.3000
5.02	4.9846	0.0	0.5936	0.4000
6.27	1.7390	0.0	0.2071	0.5000
7.52	0.4801	0.0	0.0572	0.6000
8.78	0.1049	0.0	0.0125	0.7000
10.03	0.0181	0.0	0.0022	0.8000
11.29	0.0025	0.0	0.0003	0.9000
12.54	0.0005	0.0	0.0001	1.0000

TRANSECT 4 BETA= 0.000800 RKS= 0.0

X	BW	HW	UW
2240.000	58.589	0.547	0.391

QY	C(X,QY)	CUI	C/CA	QY/QT
0.0	27.0891	0.0	3.2260	0.0
1.25	24.9630	0.0	2.9728	0.1000
2.51	19.5345	0.0	2.3263	0.2000
3.76	12.9812	0.0	1.5459	0.3000
5.02	7.3253	0.0	0.8724	0.4000
6.27	3.5103	0.0	0.4180	0.5000
7.52	1.4285	0.0	0.1701	0.6000
8.78	0.4936	0.0	0.0588	0.7000
10.03	0.1451	0.0	0.0173	0.8000
11.29	0.0375	0.0	0.0045	0.9000
12.54	0.0153	0.0	0.0018	1.0000

TRANSECT 5 BETA= 0.001000 RKS= 0.0

X	BW	HW	UW
3398.000	57.320	0.561	0.390

QY	C(X,QY)	CUI	C/CA	QY/QT
0.0	19.4579	0.0	2.3172	0.0
1.25	18.6544	0.0	2.2215	0.1000
2.51	16.4375	0.0	1.9575	0.2000
3.76	13.3127	0.0	1.5854	0.3000
5.02	9.9100	0.0	1.1802	0.4000
6.27	6.7813	0.0	0.8076	0.5000
7.52	4.2684	0.0	0.5083	0.6000
8.78	2.4797	0.0	0.2953	0.7000
10.03	1.3538	0.0	0.1612	0.8000
11.29	0.7574	0.0	0.0902	0.9000
12.54	0.5736	0.0	0.0683	1.0000

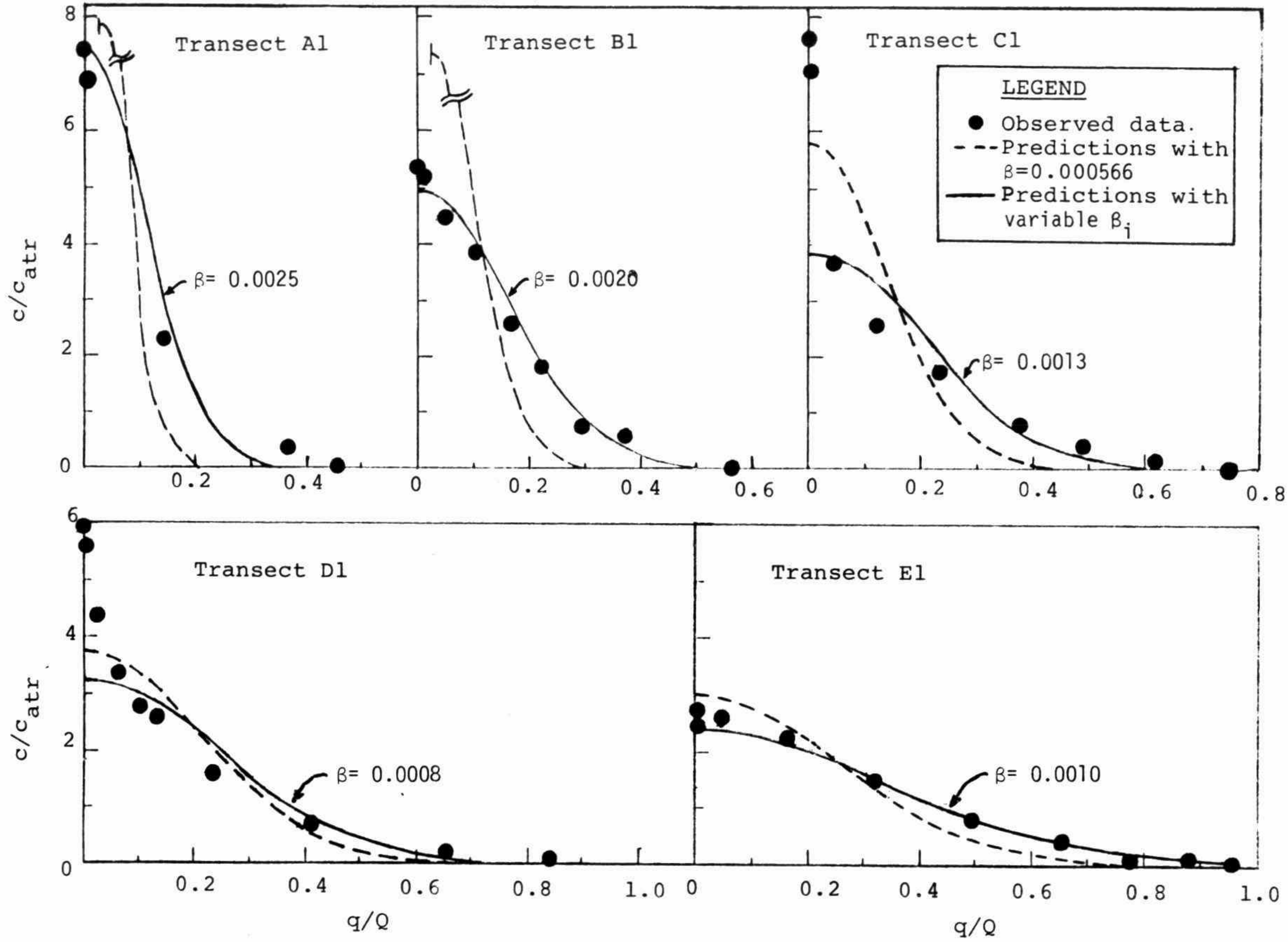


FIGURE 16. MODEL CALIBRATION RESULTS - GRAND RIVER

test conditions. The model input parameters c_e, Q_e, Q and the channel hydraulic parameters at the transects A2, B2, C2, D2 and E2 were obtained from the August test data. The values of β_i were the same as those obtained during the calibration study. Table 6 shows the predictions for the August test conditions. Dimensionless plots of the observed and the predicted distributions are shown in Fig. 17. At Transects C2 and D2, the distributions near q = 0 show the same patterns as those of the June test. The agreement at Transect E2 is not as good as that of the calibration study, possibly due to the differing locations of E1 and E2; however, the difference is not too large. The verification results are considered to be satisfactory on an overall basis.

4.4 Prediction of Ammonia

4.4.1 Water Quality Criterion for Ammonia:

In aqueous solutions, ammonia is present in ionized (NH_4^+) and un-ionized (NH_3) forms, the fraction of each being dependent on pH and temperature. The latter form is also known as toxic ammonia because of its potential to be toxic to aquatic biota. According to the MOE surface Water Quality Objectives (18), the instream criterion for un-ionized ammonia, c_s, is 0.02 mg/L (as N) for the protection of aquatic biota. The corresponding concentration of total ammonia, c_{st}, at known temperature and pH values can be calculated from $c_{st} = c_s/R''$, where R'' is given by Eq. 7 (see Chapter II). Generally, increases in temperature and pH result in an increase in the fraction of un-ionized ammonia with a corresponding decrease in the ionized fraction and vice versa.

4.4.2 Management Options:

The spatial distributions of total and un-ionized ammonia in the mixing zone of the Grand River below the Waterloo WPCP outfall were predicted under several management options using the computer program MIXAPPLN. The options considered in this study are as follows:

TABLE 6. MODEL VERIFICATION RUN- GRAND RIVER

WATERLOO AUGUST 12, 1975 TEST * CHLORIDE DATA *

TRANSECT: 1 BETA= 0.002500 RKS= 0.0

X	BW	HW	UW
183.000	64.010	0.540	0.290

QY	C(X,QY)	CUI	C/CA	QY/QT
0.0	80.4254	0.0	6.6735	0.0
1.01	56.6872	0.0	4.7038	0.1000
2.02	19.8501	0.0	1.6471	0.2000
3.03	3.4532	0.0	0.2865	0.3000
4.04	0.2984	0.0	0.0248	0.4000
5.05	0.0128	0.0	0.0011	0.5000
6.06	0.0003	0.0	0.0000	0.6000
7.07	0.0000	0.0	0.0000	0.7000
8.08	0.0000	0.0	0.0000	0.8000
9.09	0.0000	0.0	0.0000	0.9000
10.10	0.0000	0.0	0.0000	1.0000

TRANSECT: 2 BETA= 0.002000 RKS= 0.0

X	BW	HW	UW
533.000	70.015	0.561	0.257

QY	C(X,QY)	CUI	C/CA	QY/QT
0.0	55.1039	0.0	4.5724	0.0
1.01	46.7596	0.0	3.8800	0.1000
2.02	28.5718	0.0	2.3708	0.2000
3.03	12.5713	0.0	1.0431	0.3000
4.04	3.9829	0.0	0.3305	0.4000
5.05	0.9087	0.0	0.0754	0.5000
6.06	0.1493	0.0	0.0124	0.6000
7.07	0.0177	0.0	0.0015	0.7000
8.08	0.0015	0.0	0.0001	0.8000
9.09	0.0001	0.0	0.0000	0.9000
10.10	0.0000	0.0	0.0000	1.0000

TRANSECT: 3 BETA= 0.001300 RKS= 0.0

X	BW	HW	UW
1082.000	66.194	0.558	0.273

QY	C(X,QY)	CUI	C/CA	QY/QT
0.0	46.6434	0.0	3.8703	0.0
1.01	41.4663	0.0	3.4408	0.1000
2.02	29.1348	0.0	2.4175	0.2000
3.03	16.1785	0.0	1.3425	0.3000
4.04	7.1003	0.0	0.5892	0.4000
5.05	2.4628	0.0	0.2044	0.5000
6.06	0.6751	0.0	0.0560	0.6000
7.07	0.1463	0.0	0.0121	0.7000
8.08	0.0250	0.0	0.0021	0.8000
9.09	0.0034	0.0	0.0003	0.9000
10.10	0.0007	0.0	0.0001	1.0000

TRANSECT: 4 BETA= 0.000800 RKS= 0.0

X	BW	HW	UW
2240.000	56.791	0.537	0.331

QY	C(X,QY)	CUI	C/CA	QY/QT
0.0	38.2769	0.0	3.1761	0.0
1.01	35.3613	0.0	2.9342	0.1000
2.02	27.8806	0.0	2.3135	0.2000
3.03	18.7611	0.0	1.5567	0.3000
4.04	10.7746	0.0	0.8940	0.4000
5.05	5.2811	0.0	0.4382	0.5000
6.06	2.2092	0.0	0.1833	0.6000
7.07	0.7888	0.0	0.0654	0.7000
8.08	0.2407	0.0	0.0200	0.8000
9.09	0.0651	0.0	0.0054	0.9000
10.10	0.0277	0.0	0.0023	1.0000

TRANSECT: 5 BETA= 0.001000 RKS= 0.0

X	BW	HW	UW
3429.000	52.689	0.520	0.369

QY	C(X,QY)	CUI	C/CA	QY/QT
0.0	26.6526	0.0	2.2116	0.0
1.01	25.6482	0.0	2.1282	0.1000
2.02	22.8565	0.0	1.8966	0.2000
3.03	18.8626	0.0	1.5652	0.3000
4.04	14.4163	0.0	1.1962	0.4000
5.05	10.2062	0.0	0.8469	0.5000
6.06	6.7001	0.0	0.5560	0.6000
7.07	4.0980	0.0	0.3400	0.7000
8.08	2.3860	0.0	0.1980	0.8000
9.09	1.4422	0.0	0.1197	0.9000
10.10	1.1441	0.0	0.0949	1.0000

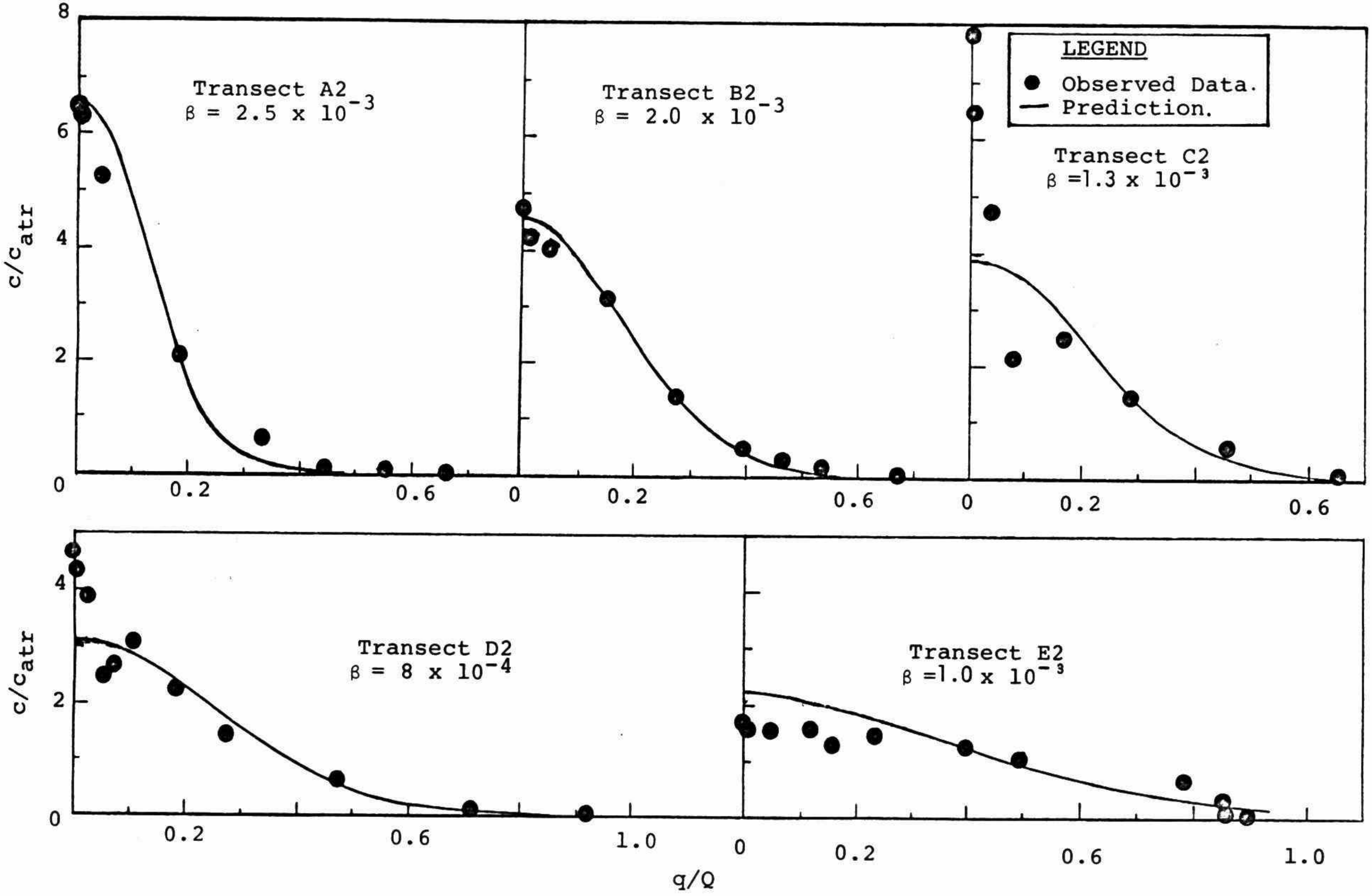


FIGURE 17. MODEL VERIFICATION RESULTS - GRAND RIVER

(i) Two effluent flow rates, 0.307 and 0.60 m^3/s, representing the present and a future hydraulic capacity of the plant.

(ii) Two river discharge values (just upstream of the outfall), 4.24 and 8.49 m^3/sec, attainable <u>without</u> and <u>with</u> flow augmentation from existing reservoirs.

(iii) Three temperature values, 15°, 20° and 25°C, reflecting the range of seasonal variability during the open water season (May-October).

(iv) Two pH values, 7.8 and 8.3 units, which are also based on observed seasonal variability of this parameter in the Grand River.

There are 24 combinations of the parameters associated with the above options, requiring 24 separate runs of the model. The options listed above are extracted from a technical report prepared by Post and Gowda (17), which deals with effluent mixing zone studies undertaken as part of the Grand River Basin Water Management Study (GRBWMS).

4.4.3 Input Data for MIXAPPLN Program:

The channel hydaulic parameters (viz. river discharge, width, average depth and mean velocity) obtained from the June 1975 field test, were used as reference values. The width, depth and velocity parameters were scaled-up for the design streamflow conditions for each option listed above using the Leopold-Maddock equations (Eqs. 38-40) in which the values of the exponents were: bex = 0.05, hex = 0.50 and uex = 0.45. The background concentration of total ammonia in the river (just upstream of the Waterloo WPCP outfall) was assumed to be zero. The Kitchener WPCP outfall, located 20,487 meters below the Waterloo outfall, was considered to be the limiting downstream boundary.

The concentration of total ammonia in the Waterloo WPCP effluent is taken to be 12.5 mg/L (as N), and is representative of the concentration present in conventional activated sludge plant effluents. (Note: The plant was producing nitrified effluent at the time of the field studies in 1975, although it was classified as a conventional activated sludge plant). The instream decay rate of total ammonia was taken as 2.31×10^{-5}/sec or 2.0/day (to base e) at 23^{o}C, based on previous waste assimilation survey results; the decay rate was assumed to be constant in the entire stretch of the river under study. The temperature correction coefficient was 1.106 for the scale-up of the decay rate to the design temperature conditions.

4.4.4 Modelling Results:

Table 7 shows a summary of the modelling results for the 24 options stated above (see subsection 4.2.2). (Note: These results can also be used to examine the sensitivity of the model to changes in various design parameters). The input parameters Q_e, Q, temperature and pH are listed in columns 2 to 5. The instream average concentration at the downstream boundary (i.e. just upstream of the Kitchener WPCP outfall) is given in column 6. The downstream distance along the shoreline x_{sce}, at which the concentration of total ammonia becomes c_{st} when c_e = 12.5 mg/L (as N), is listed in column 7. The next column shows the lateral boundary of the LUZ, q_L, as a fraction of the river discharge, and the last two columns include the values of allowable effluent concentration, c_{eA}, and the shoreline distance, x_{sceA}, at which the criterion is met.

The following general features, which indicate sensitivity aspects as well as the relative effects of various management options, are evident from the results presented in Table 7:

1. When c_e = 12.5 mg/L (as N), x_{sce} decreases due to a decrease in temperature, pH and Q_e, and an increase in Q.

TABLE 7. AMMONIA IN THE GRAND RIVER- SUMMARY OF PREDICTIONS

SUMMARY OF RUNS FOR MANAGEMENT OPTIONS
AMMONIA IN MIXING ZONE-GRAND RIVER BELOW WATERLOO WPCP *

RUN#	QEFL	QPUP	TEMP	PH	CAWP	XSCE	QY/QT	CEA	XSCEA
1.	0.307	4.24	15.0	7.8	0.358	5410.7	0.20	7.56	2970.8
							0.30	12.17	5037.8
							0.40	16.91	7892.2
2.	0.307	4.24	15.0	8.3	0.358	19417.0	0.20	2.48	3054.7
							0.30	3.99	5036.3
							0.40	5.54	7892.2
3.	0.307	4.24	20.0	7.8	0.205	6855.1	0.20	5.40	2789.6
							0.30	9.04	4725.1
							0.40	12.83	7040.2
4.	0.307	4.24	20.0	8.3	0.205	17219.4	0.20	1.80	2738.3
							0.30	3.01	4726.3
							0.40	4.27	7040.2
5.	0.307	4.24	25.0	7.8	0.081	6989.8	0.20	3.85	2424.2
							0.30	7.05	4234.1
							0.40	10.30	5975.0
6.	0.307	4.24	25.0	8.3	0.081	14381.8	0.20	1.31	2462.1
							0.30	2.40	4231.3
							0.40	3.50	5975.1
7.	0.307	8.49	15.0	7.8	0.231	2539.0	0.20	14.45	3143.1
							0.30	22.93	5576.7
							0.40	31.61	8539.1
8.	0.307	8.49	15.0	8.3	0.231	11203.3	0.20	4.74	3255.1
							0.30	7.52	5575.9
							0.40	10.36	8539.1
9.	0.307	8.49	20.0	7.8	0.152	3398.0	0.20	10.27	2975.5
							0.30	16.81	5040.4
							0.40	23.52	7722.0
10.	0.307	8.49	20.0	8.3	0.152	12409.3	0.20	3.42	3064.9
							0.30	5.60	5038.5
							0.40	7.83	7722.1
11.	0.307	8.49	25.0	7.8	0.076	4496.1	0.20	7.50	2784.2
							0.30	12.97	4710.3
							0.40	18.50	6813.6
12.	0.307	8.49	25.0	8.3	0.076	11843.9	0.20	2.55	2733.3
							0.30	4.41	4711.7
							0.40	6.29	6813.6
13.	0.600	4.24	15.0	7.8	0.674	11621.9	0.20	4.11	3088.2
							0.30	6.61	5062.0
							0.40	9.19	7964.7
14.	0.600	4.24	15.0	8.3	0.674	-999.0	0.20	1.35	2967.0
							0.30	2.17	5064.2
							0.40	3.01	7964.7
15.	0.600	4.24	20.0	7.8	0.391	12154.2	0.20	2.94	2774.5
							0.30	4.91	4756.6
							0.40	6.95	7106.5
16.	0.600	4.24	20.0	8.3	0.391	-999.0	0.20	0.98	2804.3
							0.30	1.63	4755.9
							0.40	2.31	7106.5
17.	0.600	4.24	25.0	7.8	0.159	11042.3	0.20	2.09	2395.0
							0.30	3.78	4222.3
							0.40	5.57	6034.3
18.	0.600	4.24	25.0	8.3	0.159	19260.5	0.20	0.71	2470.5
							0.30	1.28	4216.9
							0.40	1.89	6034.5
19.	0.600	8.49	15.0	7.8	0.441	5738.6	0.20	7.64	3280.8
							0.30	12.11	5592.1
							0.40	16.69	8570.9
20.	0.600	8.49	15.0	8.3	0.441	-999.0	0.20	2.50	3271.6
							0.30	3.97	5592.2
							0.40	5.47	8570.9
21.	0.600	8.49	20.0	7.8	0.293	7818.8	0.20	5.42	2961.9
							0.30	8.87	5056.1
							0.40	12.42	7763.0
22.	0.600	8.49	20.0	8.3	0.293	-999.0	0.20	1.81	2962.3
							0.30	2.95	5056.0
							0.40	4.13	7763.0
23.	0.600	8.49	25.0	7.8	0.148	8539.7	0.20	3.96	2761.0
							0.30	6.85	4733.7
							0.40	9.75	6849.6
24.	0.600	8.49	25.0	8.3	0.148	17915.6	0.20	1.35	2718.5
							0.30	2.33	4734.8
							0.40	3.31	6849.6

2. The concentration of total ammonia at the downstream boundary (i.e., just upstream of the Kitchener outfall) decreases due to increases in temperature and Q, and a decrease in Q_e.

3. An increase in the lateral boundary of the LUZ alone results in increases in both c_{eA} and x_{sceA}.

4. A sole increase in pH would decrease c_{eA}; the values of x_{sceA} are not affected by pH. (Note: Some values of x_{sceA} are seen to differ due to a change in pH only; this is due to the 5% difference permitted in the iterative procedure).

5. As temperature increases, both c_{eA} and x_{sceA} decrease.

6. An increase in the river discharge, Q, would result in increased c_{eA} and x_{sceA}.

7. As Q_e increases, c_{eA} decreases and x_{sceA} increases. (Note: In some cases, the values of x_{sceA} are seen to decrease with an increase in Q_e; this may be due to the 5% difference permitted in the iterative procedure.)

It should be noted that the foregoing salient features are of a general nature valid for the case of a nonconservative material following first order decay kinetics (viz., chlorine, phenol, etc.); however, the features involving pH effects are specific to the case of ammonia only.

V CASE STUDY II - RESIDUAL CHLORINE IN THE BOYNE RIVER BELOW ALLISTON

5.1 Water Quality Criterion for Chlorine

Chlorine is widely used as a disinfecting agent in wastewater treatment practice in order to destroy microorganisms of public health concern. In recent years, the discharge of chlorinated effluents into streams, rivers and lakes has caused concern because of the potential toxicity of chlorine to aquatic biota. The residual chlorine is mostly in the combined form (i.e., chloramines), and free residuals (i.e., hypochlorite ion and hypochlorous acid) are seldom present in treated sewage effluents. Usually, the toxicity of residual chlorine to aquatic biota is expressed in terms of total residual chlorine (TRC), representing the sum of the two forms of the residual. U.S. Environmental Protection Agency criteria for TRC are 0.002 mg/L for salmonid fish and 0.01 mg/L for other fresh water and marine organisms (24); the MOE Water Quality Objective for TRC is 0.002 mg/L (19). A review of the literature indicates that the toxicity of chlorine is a function of temperature, pH and dissolved oxygen (3, 4); however, the foregoing TRC criteria ignore the effects of these parameters.

5.2 Description of Field Studies

The study stretch selected for this case study is a 2.9 km segment of the Boyne River located between the Alliston WPCP outfall and the confluence of the Boyne with the Nottawasaga River. The mean annual daily flow in the Boyne River upstream of the Alliston WPCP is 1.62 m^3/sec and the minimum daily flow is 0.11 m^3/sec (based on 1968-70 data). Effluent from a food processing industry (Salada Foods) is discharged into the creek at about 945 m below the Alliston WPCP outfall. A schematic layout of the study stretch of the river is shown in Fig. 18.

During the field investigations, data on channel geometry, flow rates, time of travel and effluent dispersion characteristics were

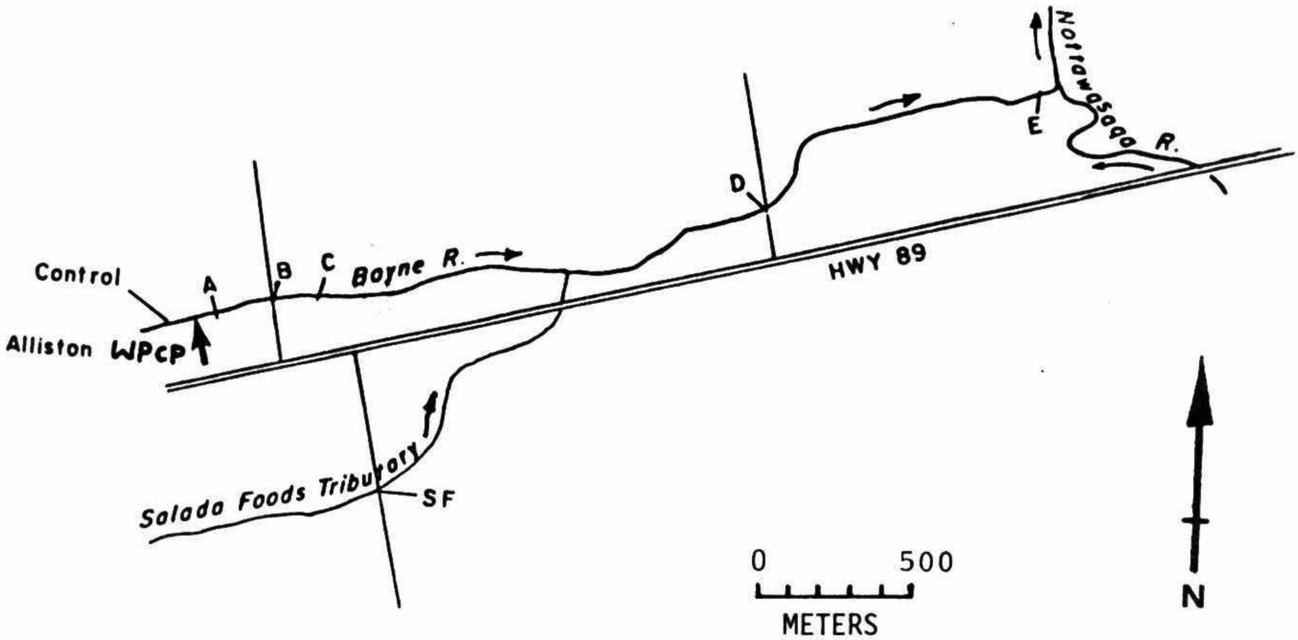


FIGURE 18. SCHEMATIC LAYOUT OF STUDY AREA - BOYNE RIVER

collected. Instream residual chlorine levels at various distances below the outfall were measured during a preliminary run. Based on these data, several transects were established in the stream for sampling purposes. At some of the transects below the outfall, cross-sectional sampling points were located based on the effluent dispersion characteristics, in order to collect data on lateral variations of water quality parameters including residual chlorine concentrations. The details of field data at each location, including the number of sampling runs and parameters measured, are presented by Wisz, Ellis and Inniss (27). Some results of data analysis and modelling studies are provided in previous publications (8, 10). A brief summary of the field data is presented below.

An intensive field survey was carried out in the Boyne River during August 11-12, 1976. During the survey, the streamflow in the Boyne River upstream of the Alliston WPCP was measured at 0.78 m^3/sec and the average instream temperature was 21.5^{o}C; the average effluent flow rate was 39.6 L/sec. The average concentrations of total residual chlorine and chloride ion at various sampling points for the duration of the survey are summarized in Table 8.

5.3 Data Analysis

The details of estimation of various parameters are described in previous publications (8, 10). The salient results of data analysis are summarized below.

5.3.1 Diffusion Factor:

The simple method described in Section 3. has been used to estimate the coefficient, β. Using the chloride ion data presented in Table 8, the net maximum concentration at each transect was determined from the observed maximum concentration, minus the background value; the value of c_a is 6.33 mg/L from the data given above. The plot of $(c_a/c_{max})^2$ versus (x/b) is shown in Fig. 19, from which the value of β is found to be 0.0038, based on an estimated best-fit line. The form of the curve at very low

TABLE 8: SUMMARY OF BOYNE RIVER FIELD SURVEY DATA - AUGUST 11-12, 1976

Transect and Sampling Station	Distance below Outfall m	Transverse Distance below Outfall Bank m	Avg[a] TRC ug/L	Avg Chlorides[a] as Cl^- mg/L
Control			0	13.7
Effluent	0		1170	131.0
Transect A	21	8.84[b]	100[c]	
A½		0.381	220	47.0
A1		0.76	18	27.0
A2		1.52	0	16.2
A3		2.29	0	14.4
A4		4.88	0	13.6
A5		6.71	0	13.6
Transect B	61	10.70[b]	16[c]	
B1		1.52	23	24.3
B2		3.05	10	20.4
B3		4.57	2	17.0
B4		7.62	0	13.6
B5		9.45	0	13.4
Transect C	183	6.40[b]	3[c]	
C1		1.22	6	20.0
C2		2.44	3	19.0
C3		3.66	1.4	17.0
C4		4.88	0.6	15.0
Salada Foods Outfall	945		0	21.0
Transect D	1148		0	18.0
Transect E	2897		0	17.0

[a]Average concentration of five samples taken over a duration of 27 hours (except three samples at A½).

[b]Channel width at the transect.

[c]Mean concentration in the effluent plume.

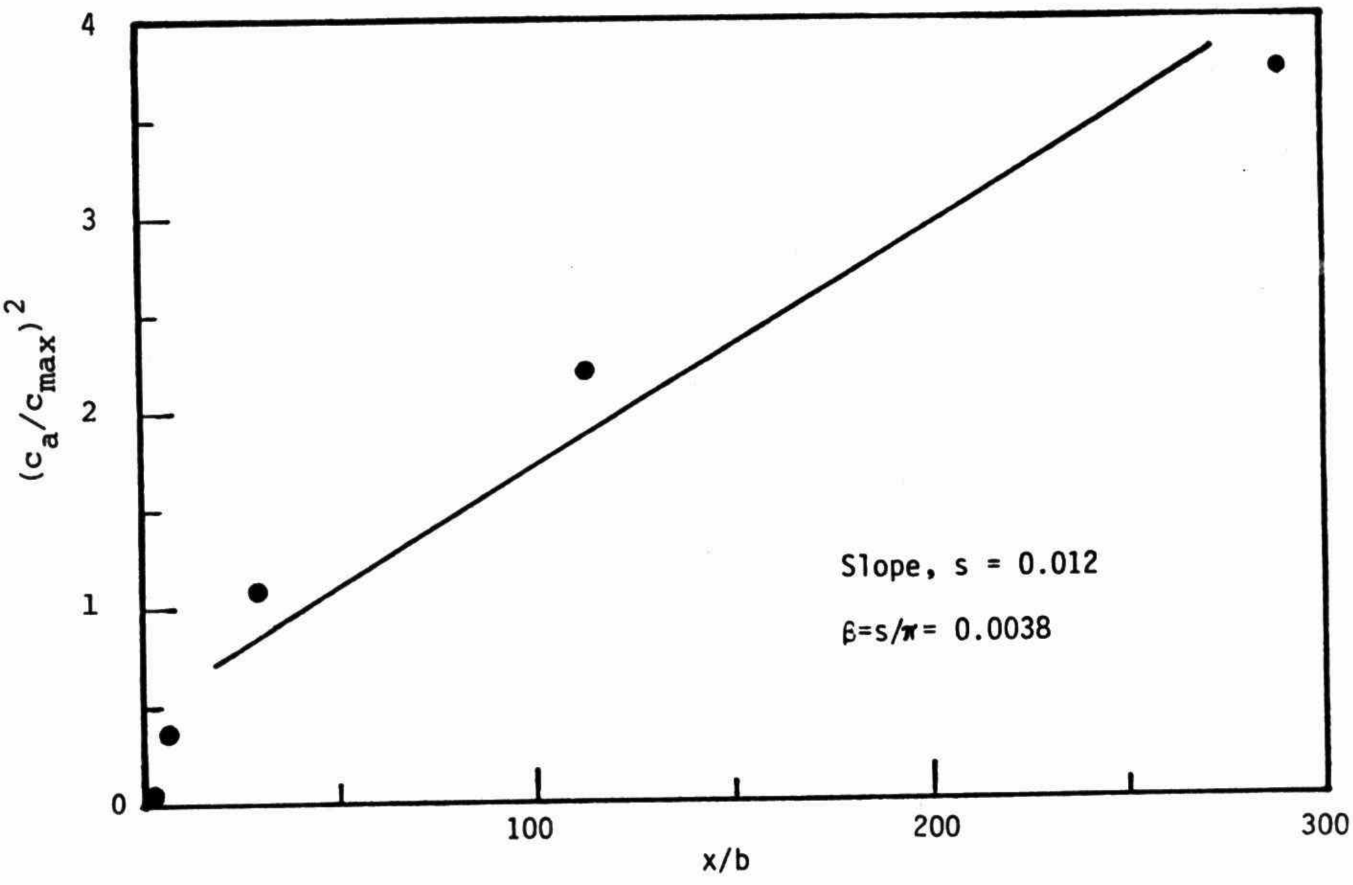


FIGURE 19. PLOT OF $(c_a/c_{max})^2$ VERSUS x/b - BOYNE RIVER

values of $(c_a/c_{max})^2$ and (x/b) is somewhat similar to that in Fig. 14(b), and is attributed to initial mixing effects; this analysis therefore considered only the linear relationship at higher values.

5.3.2. Decay Rates:

The available data on cross-sectional profiles of depth, velocity and concentration of TRC were insufficient to compute the mass flux of TRC at each transect (required to determine decay rates from a semi-logarithmic plot). Hence, an alternative method involving determination of gross decay rates (due to dispersion, chemical reaction, etc.) and apparent attenuation rates (due to dispersion only), has been used to calculate the decay rates of TRC; this method is described below.

The gross TRC decay rate coefficient, k_t, due to mixing and all other mechanisms, was determined from a semi-log plot of observed average TRC concentration in the plume vs. distance, shown in Fig. 20. An examination of the plot indicates that the rate varies significantly in two successive reaches as indicated by the slopes of the solid lines. The estimated rate coefficients, k_t, for the two reaches AB and BC are 0.0104 and 0.0031 per second (to base e), respectively, at the observed mean temperature of 21.5°C.

The width of the effluent plume at each transect was estimated from the chloride ion distributions. The partial discharge within the plume width at each transect was then calculated on the assumption that the discharge per unit width of channel is the same. Treating the TRC as a conservative material, the average TRC concentration values within the effluent plume, due to mixing effects only and computed from mass balance, were: 177.69, 112.65, 70.38 and 56.98 ug/L at Transects A, B, C and D, respectively. On the assumption that the decrease in concentration due to mixing follows an exponential law, the apparent attenuation rate coefficients due to mixing (k_p) are found to be 0.0026 and 0.00088 per second (to base e) for the reaches AB and BC, respectively (see Fig. 20), representing approximately 25% and 28% of the gross decay rate

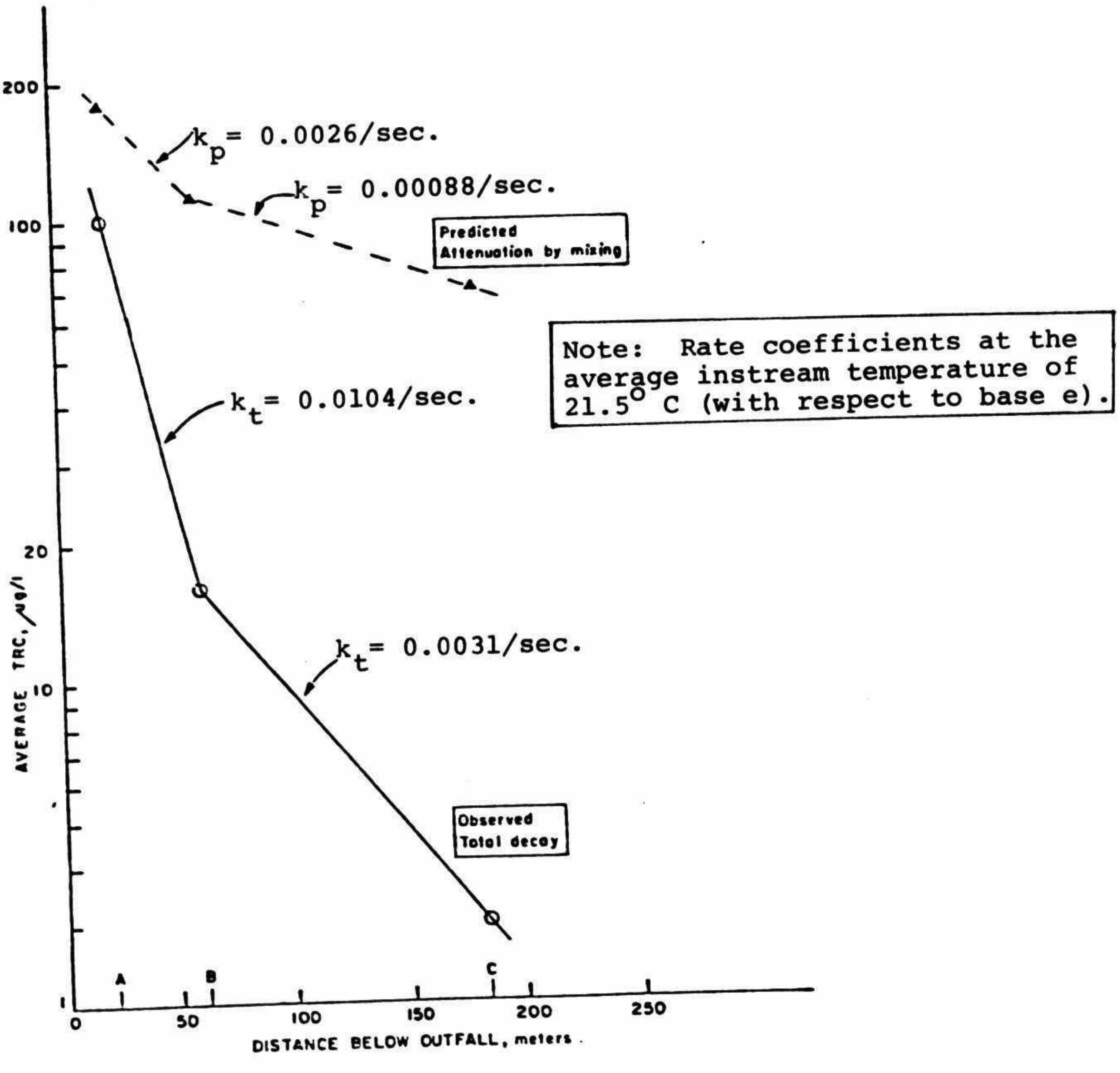


FIGURE 20. ATTENUATION AND DECAY OF TRC IN BOYNE RIVER

values. Thus, attenuation of TRC concentration due to mixing only accounts for about 30% of the observed decrease. The corresponding decay rates of TRC (k_d) attributable to chemical reactions and other mechanisms, determined from $k_d = (k_t - k_p)$, are 0.0078 and 0.0022 per second (to base e) at 21.5 ^oC. (Note: The foregoing method can be applied to a stretch below the outfall within which the effluent has not filled the entire cross-section, i.e., within the crossing distance shown in Fig.1. The mass flux method described in Section 3.2 is applicable to all cases).

5.4 Model Validation Study

Due to the limitations of data available, the model validation was carried out in one step only (instead of the calibration and verification procedures as described in Chapter IV). Using the parameters obtained from the survey data as input to the MIXCALBN program, concentration predictions were obtained with $\beta_i = 0.0038$; however, the predictions were not in agreement with the observations. Then, several runs were carried out by selecting different combinations of β and k_d for each transect; and satisfactory agreements between the observations and the predictions, shown in Fig. 21, were obtained with the following combinations:

Transect	β	k_d
A	0.0005	0.007/sec.
B	0.0025	0.007/sec.
C	0.0025	0.0022/sec.

The predictions shown in Fig. 21 are based on the approximation that the discharge per unit width of channel is the same at each transect. Even with this approximation, the predictions of the stream tube model are in good agreement with the observations. In contrast, results presented in previous publications (8, 10) show that the predictions of the models based on width as the lateral variable differ considerably from the observations at some transects.

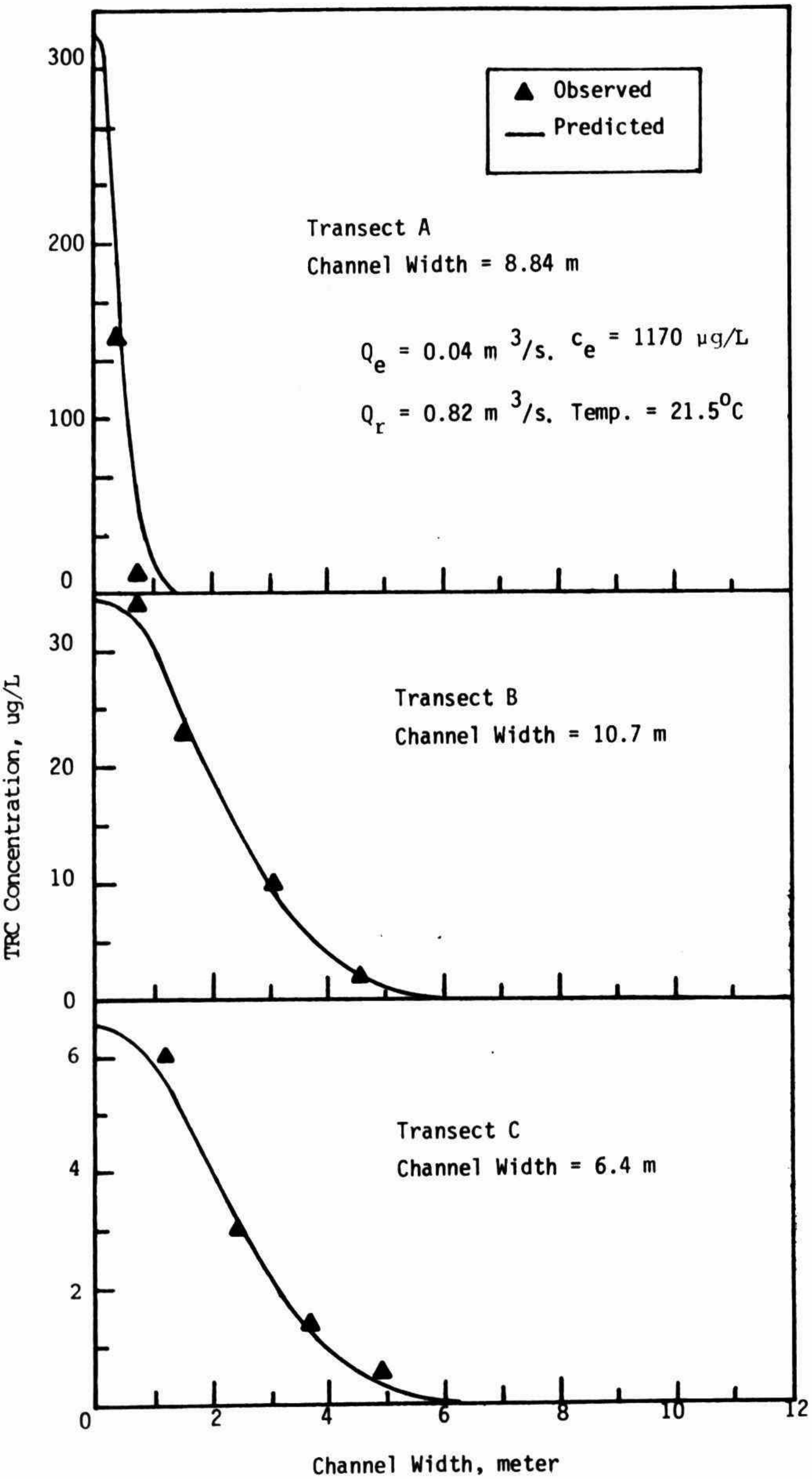


FIGURE 21. MODEL VALIDATION RESULTS - BOYNE RIVER

5.5 Critical Point Method Predictions

Using the parameters determined in the model validation step, the transverse distributions of TRC were predicted at several closely spaced distances below the outfall. From these predictions, the longitudinal distributions of TRC along $q_L = 0.25Q$ and $q_L = 0.35Q$ were plotted as shown in Fig. 22. The occurrence of the critical point is clearly demonstrated in these plots. The critical concentrations are seen to be higher than the MOE objective of 2 ug/L. For comparison purposes, the values of TRC at the appropriate values of q_L interpolated from the observed cross-sectional distributions at Transects B and C are also shown on the plots. The curves shown in Fig. 22 exhibit steep changes in slope at some points, indicating the effects of variations in k_d along the river stretch.

For the same design conditions as above - viz., c_e = 1170 ug/L, c_s = 2 ug/L, $Q_e = 0.04\ m^3/s$, and $Q = 0.82\ m^3/s$, the critical co-ordinates and allowable effluent TRC concentrations were calculated by the graphical method described in Section 2.5, using estimated average values $\beta = 0.0025$, $k_{av} = 0.007$ per second at 21.5°C (to base e), B = 9.9 m, and U = 0.226 m/s. The completely mixed instream average concentration, c_a = 57.07 ug/L. The results for p_L (i.e., q_L/Q) = 0.25 and p_L = 0.35 are given in Table 9. For comparison purposes, the values of x_L, c_L and c_{eA} predicted from the MIXAPPLN program are also given in Table 9 (inside parentheses). The predictions of the two methods are seen to be in reasonable agreement. (Note: The graphical procedure may involve some approximations due to the need to use constant values of B, U and k_{av}, and thus it is more suitable for estimation of the critical co-ordinates during the preliminary planning stages of a project, as stated earlier in this report.)

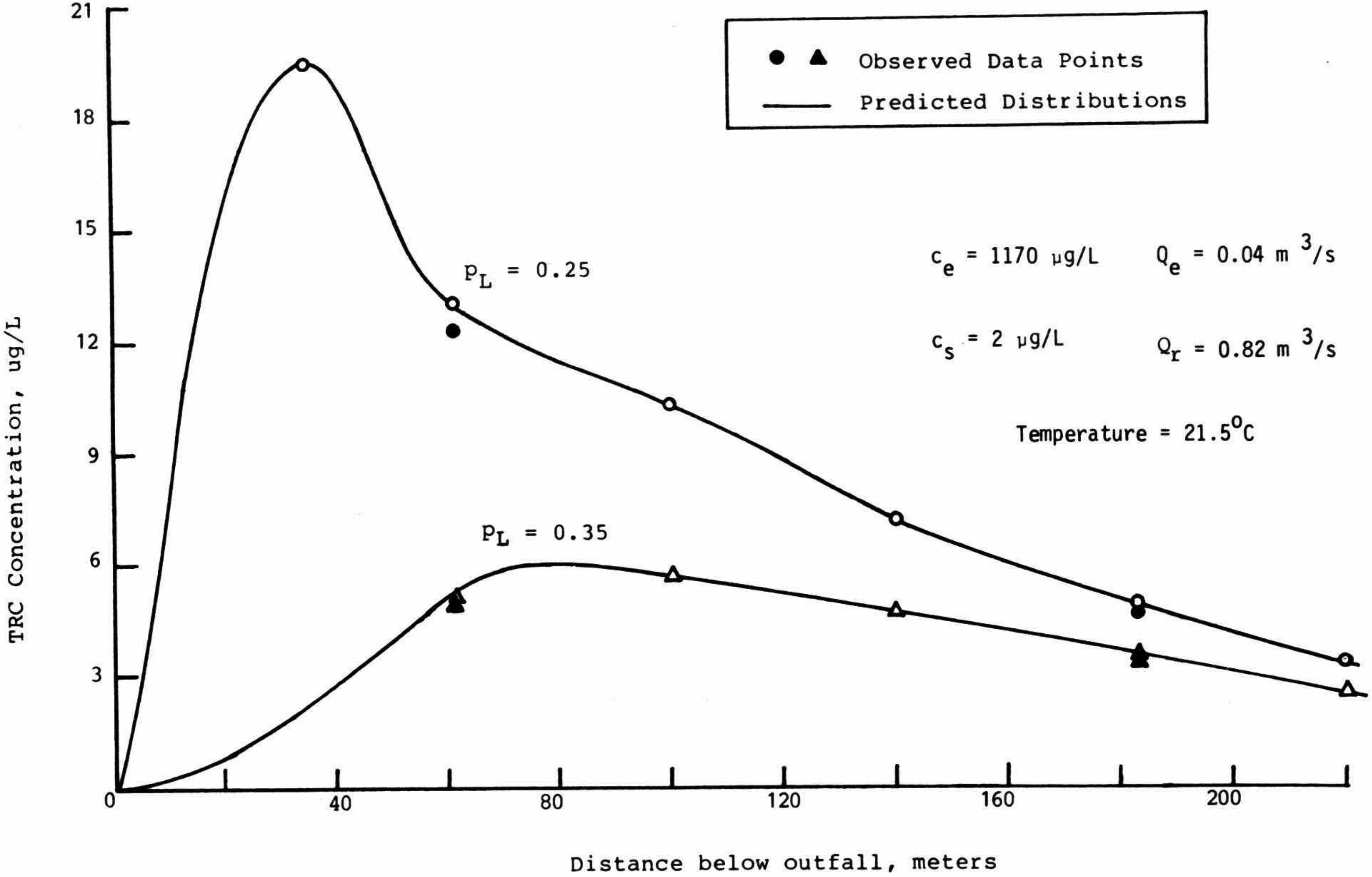


FIGURE 22. DISTRIBUTION OF TRC ALONG THE LATERAL BOUNDARY OF LUZ IN THE BOYNE RIVER

TABLE 9: RESULTS OF CRITICAL POINT METHOD

p_L	E_L	G_L	c_r	x_L m	c_L ug/L	c_{eA} ug/L	% reduction = $100(c_e - c_{eA})/c_e$
0.25	122.7	1.2	0.30	38.7 (35.0)	17.1 (19.5)	136.8 (120.0)	88.3 (89.7)
0.35	122.7	1.7	0.088	54.9 (57.0)	5.0 (6.0)	468.0 (390.0)	60.0 (66.7)

With the design parameters listed in the previous paragraph, computations based on the instantaneous complete mixing (ICM) assumption would result in an allowable effluent TRC concentration of 41 ug/L, requiring a reduction of 96.5%. In contrast, the design based on the LUZ concept with p_L = 0.35 requires a reduction of about 60% only. The results of examples presented in another publication (11) indicate that in some cases, the ICM assumption or the dilution ratio concept may lead to a LUZ with too large a lateral boundary (and consequently, too small a zone of passage) due to an underestimation of treatment required in comparison to the predictions based on the LUZ concept; whereas, in some other situations, the predictions based on the former method may result in treatment requirements that are too stringent in comparison to those of the latter method. Thus, in the case of discharge from a bank outfall, it is inappropriate to determine the allowable effluent concentration of a pollutant based on the instantaneous complete mixing assumption.

VI SUMMARY AND CONCLUSIONS

This report presents the development and application of steady state mathematical models to predict concentration distributions of nonconservative pollutants (subjected to first order decay) in the mixing zones of shallow rivers receiving effluents from bank outfalls. The models are based on the "stream tube" concept introduced by Yotsukura and his co-workers, wherein the lateral variable is taken to be the partial cumulative discharge (instead of the lateral distance). An analytical solution of the convection-diffusion equation presented by Yotsukura and Cobb (29) has been modified to account for longitudinal variabilities in decay rates and channel hydraulic parameters. An alternative analytical solution, applicable to mixing zones below bank outfalls, was obtained by following Rood and Holley (21), from which expressions have been derived for the co-ordinates of a critical point occurring in the mixing zone. Graphical procedures were also developed by writing the analytical expressions in dimensionless terms. However, the graphical procedures are primarily applicable to situations where decay rates and channel hydraulic parameters remain constant in the study stretch. When these parameters are reach-dependent, their weighted mean values can be used in the graphical method. A simple expression for computing allowable effluent concentration (to meet a specified instream criterion at a known lateral boundary of a limited use zone) has been derived. A simple method of incorporating the effects of background concentrations is also described.

An iterative procedure is outlined to predict the co-ordinates of critical point (by interpolation or extrapolation) when the instream model parameters are reach-dependent. Also computed by iteration are the allowable effluent concentration and the maximum longitudinal distance along the outfall shoreline at which the criterion concentration is met.

Two computer programs have been developed to predict concentration distributions in mixing zones by using the modified stream tube model equation. One of the programs, MIXCALBN, is suitable for model validation studies and the other program, MIXAPPLN, is useful to evaluate various management options (viz. streamflow and temperature variations, treatment plant expansion, etc.). These two models are documented in Appendix A.

General procedures are outlined for carrying out field surveys. The data are utilized to calculate mass flux values, mean depths, mean velocities, shape-velocity factors and variance of tracer distributions; a computer program MIXANDAT, documented in Appendix B, is utilized in the analysis of data.

The relationships of variance of a tracer distribution curve to dispersion coefficient and diffusion factor were modified to account for the effects of variabilities in width, depth and velocity, normally observed in a given stretch of a natural river. The modified relationships, written in dimensionless form, permit direct evaluation of nondimensional coefficients from graphical plots. Also developed is a simple graphical method of estimating a nondimensional coefficient (which may be termed a "dimensionless diffusion factor") from peak concentrations measured at various transects.

Details of field studies on effluent dispersion in the Grand River below the Waterloo WPCP outfall have been described. During these studies Rhodamine WT dye injected continuously at the outfall, as well as chloride ion and conductivity normally present in water and wastewater, were used as tracers. The distributions of all three tracers were utilized to evaluate dispersion characteristics.

The modified stream tube model was calibrated and verified satisfactorily using data on the distribution of chloride ion in the Grand River below the Waterloo wastewater treatment plant effluent outfall. The validated model was then applied to predict the allowable effluent concentration of total ammonia as well as maximum

longitudinal spread for given lateral boundaries of the limited use zone, under several management options. The results will form valuable input for management planning.

The model was validated satisfactorily for the case of total residual chlorine, using field data from the Boyne River below the Alliston wastewater treatment plant outfall. For this case, predictions were also made by the critical point method.

The general conclusions resulting from this study are as follows:

1. Chloride ion and specific conductance can be considered as conservative tracers to study the dispersion characteristics of a river or stream.

2. It is more appropriate to relate the transverse dispersion coefficient to width (rather than depth) in shallow streams.

3. The steady state stream tube model, based on modified analytical solutions of the 2-D convection-dispersion equation, can be applied to natural streams and rivers in which tracer and pollutant concentration profiles are continuous at successive transects.

4. The critical point method is suitable for predictive purposes during the preliminary planning stages of a project, since the method assumes that the instream process parameters are not reach-dependant.

5. An increase in the lateral boundary of a limited use zone results in an increase in the allowable effluent concentration; however, the associated longitudinal spread along the discharge shoreline would also increase.

6. When the discharge in a river is increased, the allowable effluent concentration as well as the maximum longitudinal spread would increase.

7. As temperature increases, the allowable effluent concentration and the maximum longitudinal spread decrease.

8. In the case of discharge from a bank outfall, it is inappropriate to determine the allowable effluent concentration of a pollutant based on the instantaneous complete mixing assumption (as well as the dilution ratio, streamflow rate/effluent flow rate) due to the following reasons: (a) in some cases it may lead to a LUZ with too large a lateral boundary (and consequently, too small a zone of passage) due to an underestimation of treatment required; and (b) in some other cases, it may result in too small a LUZ due to the prediction of too stringent treatment requirements in comparison to those based on the LUZ concept.

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APPENDIX A

DOCUMENTATION OF MIXCALBN AND MIXAPPLN PROGRAMS

A.1 Description of Variable Names

A.2 Data Input Aspects

A.3 List of MIXCALBN Program

A.4 Example and Input/Output of MIXCALBN Program

A.5 List of MIXAPPLN Program

A.6 Typical Input/Output of MIXAPPLN Program

A.1: DESCRIPTION OF VARIABLE NAMES

TITLE:	title of study (river, date of survey, etc.)
QRS:	reference streamflow rate just below outfall
BPWR:	width exponent, bex
HPWR:	depth exponent, hex
UPWR:	velocity exponent, uex
TEMPS:	reference temperature at which decay rates are known, ^{o}C
NTR:	number of transects
NYZ:	number of elemental strips at a transect
NQ:	number of lateral data points; = NYZ + 1
X:	longitudinal distance below outfall to a given transect, i
BS:	reference width at transect, i (at flow = QRS)
HS:	reference mean depth at transect, i (at flow = QRS)
US:	reference mean velocity at transect, i (at flow = QRS)
BETA:	nondimensional coefficient, β_i
MQ:	number of upstream flows for design cases
QRUP:	streamflow rates just upstream of outfall for the design cases
MT:	number of design case temperatures
TMP:	temperature of river water for design cases, ^{o}C
MF:	number of design case effluent flows
QEFL:	design case effluent flow rates
MPH:	number of design case pH values
PH:	pH of stream water for design cases
QCP:	partial cumulative discharge between bank and outfall location in the cross-section (equal to zero for a bank outfall)
CEFL:	effluent concentration of pollutant for design case
CS:	specified water quality objective
THETA:	temperature correction coefficient, θ

CBKG:	background concentration in river water
RBK:	decay rate coefficient associated with CBKG at the temperature, TEMPS (per second, to base e)
XWCP:	longitudinal distance to limiting downstream boundary
RKS:	decay rate coefficient for reach, i (per second, to base e) at the reference temperature, TEMPS
RF:	product function of decay terms
EY:	transverse dispersion coefficient
BW, HW, UW:	weighted mean values of width, depth and velocity for the stretch of river between the outfall and transect, i
QY:	partial cumulative discharge, q
QT:	total river discharge, Q
QY/QT:	lateral boundary of LUZ, expressed as (partial cumulative discharge/total discharge)
C(X,QY):	concentration at a point (x, q)
CUI:	concentration of un-ionized ammonia; (for pollutants other than ammonia, CUI values are set to zero).
CA:	completely mixed instream concentration just below outfall, c_a
C/CA:	ratio, C(X,QY)/CA
CAWP:	cross sectional average concentration of pollutant at the downstream boundary
CEA:	allowable concentration of pollutant in effluent
CEFL:	concentration of pollutant in effluent
CS:	specified instream water quality objective
XL:	distance below outfall to the critical point
CL:	critical concentration
XSCE:	maximum longitudinal spread along shoreline at which concentration = c_s when effluent concentration is c_e
XSCEA:	maximum longitudinal spread along shoreline at which concentration = c_s when effluent concentration is c_{eA}.
NCHNG:	a number input to select a desired option to change parameters
ITRN:	number of iterations

A.2: DATA INPUT ASPECTS

The MIXCALBN and MIXAPPLN programs are operational on time-sharing computer terminals, as stated elsewhere, in this report. The data input list of each program (included in the input/output sections) indicates the order in which the data is read by that program and should be followed in the preparation of input data.

The data can be input interactively in a "question-and-answer" session by typing in the required data in response to instructions and questions printed out during a run. Alternatively, the data can be stored on an auxiliary device such as a disk or a tape and accessed during program execution. (Note: The input data listed in Sections A.4 and A.6, were stored on disks and read during execution of the programs.)

There are options built into the program to change the model parameters or to end a run, each option being identified by a number. At the end of each run, the available options and the associated identification numbers are printed out so that a user can select a desired option. (Note: The line "TO CHANGE PARAMETERS ...", is shown on the data input lists for convenience; however, that line and the following one, are printed out at the end of each run).

A.2: LIST OF MIXCALBN PROGRAM

```
C       PROGRAM NAME: MIXCALBN  * * STREAMTUBE MODEL FOR PIPE OUTFALL
C        DEVELOPED BY T. P. H. GOWDA, WATER RESOURCES BRANCH, MOE.
C       THIS PROGRAM PREDICTS LAT'L & LONG'L DISTRN. OF CONSERVATIVE
C       AND NONCONSERVATIVE MATERIALS DISCHARGED INTO A RIVER FROM
C        A PIPE OUTFALL LOCATED AT BANK OR IN RIVER(VERT. LINE SOURCE).
C
C    DECLARATION STATEMENTS
C
        DIMENSION TITLE(20),C(10,50),CUI(10,50)
        REAL*8 X(10),XX(10),P1,P2,P3,P4,T1,T2,T3,T4,A3,RKS(10),QY(50),
       *THETA,BPWR,HPWR,UPWR,QRTO,QT,RBT,QRS,QRUP,QEFL,RBK,CTDP,PHDR,
       *RF(10),BS(10),HS(10),US(10),BETA(10),BW,HW,UW,RKT,R,PHI,TMP,
       *B(10),H(10),U(10),BSUM(10),TOT(10),VOL(10),TEMPS,PAX1,PAX2
        CALL ERRSET(208,999,-1)
C    INPUT DATA
        WRITE(6,2)
        READ(1,3) TITLE
     35 WRITE(6,4)
        READ(1,*) QRS,BPWR,HPWR,UPWR,THETA,TEMPS,RBK
     43 WRITE(6,19)
        READ(1,*)QRUP,QEFL,CEFL,CBKG,TMP
     33 WRITE(6,5)
        READ(1,*) NTR,NYZ,QCP
        WRITE(6,6)
        READ(1,*) (X(I),BS(I),HS(I),US(I),I=1,NTR)
     47 WRITE(6,55)
        READ(1,*) (RKS(I),I=1,NTR)
     45 WRITE(6,8)
        READ(1,*) (BETA(I),I=1,NTR)
        WRITE(6,52)
        READ(1,*) AMONIA,PH
C    CALCULATE FLOW & TEMP'R PARAMETERS
        DELTA=0.0001
        QT=QRUP+QEFL
        QRTO=QT/QRS
        CTDP=THETA**(TMP-TEMPS)
        RBT=RBK*CTDP
        DELQ=QT/NYZ
        NQ=NYZ+1
        WRITE(6,3)TITLE
        CA=(CEFL*QEFL /QT)
        DO 12 I=1,NTR
        BSUM(I)=0.
     12 TOT(I)=0.
        DO 14 I=1,NTR
C    CALCULATE B,H,U FOR FLOW=QT, FROM LEOPOLD-MADDOCK EQNS.
        B(I)=BS(I)*QRTO**BPWR
        H(I)=HS(I)*QRTO**HPWR
        U(I)=US(I)*QRTO**UPWR
C    CALCULATE WEIGHTED MEAN VALUES BW,HW,UW FROM OUTFALL TO TRANSECT(I)
        IF(I.GE.2) GO TO 60
        XX(1)=X(1)
        BW=B(1)
        HW=H(1)
        UW=U(1)
        BSUM(1)=B(1)*XX(1)
        VOL(1)=B(1)*H(1)*XX(1)
        TOT(1)=XX(1)/U(1)
        GO TO 62
     60 I1=I-1
        XX(I)=X(I)-X(I1)
        BSUM(I)=BSUM(I1)+0.5*XX(I)*(B(I1)+B(I))
        VOL(I)=VOL(I1)+0.25*XX(I)*(B(I1)+B(I))*(H(I1)+H(I))
        TOT(I)=TOT(I1)+XX(I)/U(I)
        BW=BSUM(I)/X(I)
        HW=VOL(I)/(X(I)*BW)
        UW=QT/(BW*HW)
C    CALCULATE PRODUCT FUNCTION FOR DECAY, RF(I).
     62 RKT=CTDP*RKS(I)
        CBKX=CBKG*DEXP(-RBT*TOT(I))
        A3=(RKT*XX(I))/U(I)
        R=DEXP(-A3)
        IF(I.GE.2) GO TO 64
        RF(1)=R
        GO TO 66
     64 RF(I)=RF(I1)*R
     66 CONTINUE
        PHI=BETA(I)*X(I)/BW
        PHDR=4.0*PHI
        CRPX=0.5*CA*RF(I)/DSQRT(3.1416*PHI)
        BGX=PHI*ALOG(1./DELTA)
        SBG=SQRT(BGX)
        WRITE(6,40) I,BETA(I),RKS(I)
        WRITE(6,23) X(I),BW,HW,UW
        WRITE(6,42)
        DO 16 K=1,NQ
        QY(K)=(K-1)*DELQ
```

```
      IF (QY(K).GT.QT) QY(K)=QT
      PAX1=(QY(K)-QCP)/QT
      PAX2=(QY(K)+QCP)/QT
C  DETERMINE NO. OF IMAGES REQUIRED
      AN1=(0.5*PAX1-SBG)-0.5
      AN2=(0.5*PAX1+SBG)+0.5
      AN3=-AN2
      AN4=-AN1
      NM1=IFIX(AN1)
      NM2=IFIX(AN2)
      NM3=IFIX(AN3)
      NM4=IFIX(AN4)
      NN1=1+NM2+IABS(NM1)
      NN2=1+IABS(NM3)+IABS(NM4)
      IF(NN1.GE.NN2)NN=NN1+1
      IF(NN1.LT.NN2)NN=NN2+1
C  COMPUTATION OF CONC'N DISTR'NS.
      SUM=0.
      DO 32 J=1,NN
      N=J-1
      P1=(PAX1-2.*N)**2/PHDR
      P2=(PAX2+2.*N)**2/PHDR
      CALL PDET(P1,T1)
      CALL PDET(P2,T2)
      IF(N.LE.0) GO TO 30
      P3=(PAX1+2.*N)**2/PHDR
      P4=(PAX2-2.*N)**2/PHDR
      CALL PDET(P3,T3)
      CALL PDET(P4,T4)
      GO TO 32
   30 T3=0.
      T4=0.
   32 SUM=SUM+T1+T2+T3+T4
      C(I,K)=CRPX*SUM+CBKX
C  CALCULATE UN-IONIZED AMMONIA CONCENTRATIONS*  OPTIONAL *
      IF(AMONIA.LE.0) GO TO 16
      PKA=0.09018+2729.92/(TMP+273.2)
      PF=PKA-PH
      PCTU=1./(1.+10.**PF)
      CUI(I,K)=C(I,K)*PCTU
   16 CONTINUE
C  PRINT OUTPUT
      DO 14 K=1,NQ
      RC=C(I,K)/CA
      RQ=QY(K)/QT
      IF(AMONIA.LE.0) CUI(I,K)=0.0
      WRITE(6,25) QY(K),C(I,K),CUI(I,K),RC,RQ
   14 CONTINUE
C  CHANGE PARAMETERS
      WRITE(6,28)
      READ(1,*) NCHNG
      GO TO(45,47,33,43,35,999),NCHNG
C
C  FORMAT STATEMENTS
    2 FORMAT(/' ENTER TITLE OF STUDY')
    3 FORMAT(20A4)
    4 FORMAT(' ENTER QRS,BPWR,HPWR,UPWR,THETA,TEMPS,RBK')
    5 FORMAT(' ENTER NTR,NYZ,QCP ')
    6 FORMAT(' ENTER NTR VALUES OF X,BS,HS,US')
    8 FORMAT(' ENTER NTR VALUES OF BETA')
   19 FORMAT(' ENTER DESIGN CASE: QRUP,QEFL,CEFL,CBKG,TMP')
   23 FORMAT(2X,4(F9.3,1X)/)
   24 FORMAT(//1X,20A4)
   25 FORMAT(2X,F5.2,2X,4(F9.4,2X))
   28 FORMAT(/'  TO CHANGE PARAMETERS ENTER THE NUMBER STATED BELOW'/
     1'  BETA: 1; RKS: 2; NTR,..: 3;  QRUP,..: 4;  QRS,..:5; END: 6')
   40 FORMAT(/5X,'TRANSECT: ',I2,2X,'BETA=',F9.6,2X,'RKS=',F9.6/7X,'X',
     *9X,'BW',9X,'HW',8X,'UW')
   42 FORMAT(4X,'QY',5X,'C(X,QY)',5X,'CUI',9X,'C/CA',6X,'QY/QT'/)
   52 FORMAT(2X,'UN-IONIZED AMMONIA: ENTER 1 FOR YES, 0 FOR NO; AND ENTE
     *R PH VALUE ON THE SAME LINE')
   55 FORMAT(2X,' ENTER NTR VALUES OF RKS')
  999 STOP
      END
C
      SUBROUTINE PDET(P,T)
      REAL*8 P,T
      IF(P.GE.40.0)GO TO 10
      T=DEXP(-P)
      GO TO 12
   10 T=0.0
   12 CONTINUE
      RETURN
      END
```

A.4: EXAMPLE AND INPUT/OUTPUT OF MIXCALBN PROGRAM

Example: Based on the example cited by Verboom (25), the following data are utilized to illustrate typical input and output of the MIXCALBN program:

A conservative material discharged from a bank outfall into a rectangular river is considered.

$Q = 96.25\ m^3/s$; $b = 55.0\ m$; $h = 3.5\ m$; $u = 0.5\ m/s$;
$e_y = 0.021\ m^2/s$; $\beta = (e_y/bu) = 7.6364 \times 10^{-4}$;
Effluent flux = 0.01 kg/s; = 10^7 ug/s
Assuming $Q_e = 10.0\ m^3/s$, we get $c_e = 1000$ ug/L.

Width of each elemental strip = 2.5 m; thus, number of elemental strips is (55/2.5) = 22, and discharge through each elemental strip is (96.25/22) = 4.375 m^3/s.

It is required to predict cross-sectional concentration distributions at two transects located 75 m and 675 m below the outfall.

(Note: In this example, the input and output concentrations are expressed in ug/L, whereas the values given by Verboom (25) are in kg/m^3; the relation between these units is 1 $kg/m^3 = 10^6$ ug/L.)

Input Data:

The input data for the above example were read from a disk. The variables and their values are listed below:

```
ENTER TITLE OF STUDY
ENTER QRS,BPWR,HPWR,UPWR,THETA,TEMPS,RBK
ENTER DESIGN CASE: QRUP,QEFL,CEFL,CBKG,TMP
ENTER NTR,NYZ,QCP
ENTER NTR VALUES OF X,BS,HS,US
  ENTER NTR VALUES OF RKS
ENTER NTR VALUES OF BETA
 UN-IONIZED AMMONIA: ENTER 1 FOR YES, 0 FOR NO; AND ENTER PH VALUE ON THE SAME LINE

 TO CHANGE PARAMETERS ENTER THE NUMBER STATED BELOW
 BETA: 1; RKS: 2; NTR,...: 3;  QRUP,...: 4;  QRS,..: 5; END: 6


   VERBOOM'S DATA FOR CHECKING MIXCALBN PROGRAM
96.25 0 .6 .4 1.03 20 0.0
86.25 10. 1000. 0. 20.
2 22 0.0
75. 55. 3.5 .5
675. 55. 3.5 .5
0. 0.
.00076364 .00076364
0 7.0
6
```

OUTPUT OF MIXCALBN PROGRAM:

VERBOOM'S DATA FOR CHECKING MIXCALBN PROGRAM

TRANSECT: 1

X	BW	HW	UW	EY
75.000	55.000	3.500	0.500	0.0210

QY	C(X,QY)	CUI	C/CA	QY/QT
0.0	1816.4780	0.0	17.4836	0.0
4.38	1106.1323	0.0	10.6465	0.0455
8.75	249.7692	0.0	2.4040	0.0909
13.13	20.9134	0.0	0.2013	0.1364
17.50	0.6493	0.0	0.0062	0.1818
21.88	0.0075	0.0	0.0001	0.2273
26.25	0.0000	0.0	0.0000	0.2727
30.63	0.0000	0.0	0.0000	0.3182
35.00	0.0000	0.0	0.0000	0.3636
39.38	0.0	0.0	0.0	0.4091
43.75	0.0	0.0	0.0	0.4545
48.13	0.0	0.0	0.0	0.5000
52.50	0.0	0.0	0.0	0.5455
56.88	0.0	0.0	0.0	0.5909
61.25	0.0	0.0	0.0	0.6364
65.63	0.0	0.0	0.0	0.6818
70.00	0.0	0.0	0.0	0.7273
74.38	0.0	0.0	0.0	0.7727
78.75	0.0	0.0	0.0	0.8182
83.13	0.0	0.0	0.0	0.8636
87.50	0.0	0.0	0.0	0.9091
91.88	0.0	0.0	0.0	0.9545
96.25	0.0	0.0	0.0	1.0000

TRANSECT: 2

X	BW	HW	UW	EY
675.000	55.000	3.500	0.500	0.0210

QY	C(X,QY)	CUI	C/CA	QY/QT
0.0	605.4927	0.0	5.8279	0.0
4.38	573.0242	0.0	5.5154	0.0455
8.75	485.6968	0.0	4.6748	0.0909
13.13	368.7107	0.0	3.5488	0.1364
17.50	250.6888	0.0	2.4129	0.1818
21.88	152.6554	0.0	1.4693	0.2273
26.25	83.2564	0.0	0.8013	0.2727
30.63	40.6678	0.0	0.3914	0.3182
35.00	17.7915	0.0	0.1712	0.3636
39.38	6.9711	0.0	0.0671	0.4091
43.75	2.4464	0.0	0.0235	0.4545
48.13	0.7689	0.0	0.0074	0.5000
52.50	0.2164	0.0	0.0021	0.5455
56.88	0.0546	0.0	0.0005	0.5909
61.25	0.0123	0.0	0.0001	0.6364
65.63	0.0025	0.0	0.0000	0.6818
70.00	0.0005	0.0	0.0000	0.7273
74.38	0.0001	0.0	0.0000	0.7727
78.75	0.0000	0.0	0.0000	0.8182
83.13	0.0000	0.0	0.0000	0.8636
87.50	0.0000	0.0	0.0000	0.9091
91.88	0.0000	0.0	0.0000	0.9545
96.25	0.0000	0.0	0.0000	1.0000

A.5: LIST OF MIXAPPLN PROGRAM

```
C      PROGRAM NAME:  MIXAPPLN  * * STREAMTUBE MODEL FOR PIPE OUTFALL
C     THIS PROGRAM IS SET UP FOR CONSERVATIVE, NONCONSERVATIVE WITH
C     FIRST ORDER DECAY(VIZ.,RESIDUAL CHLORINE, PHENOL, RADIONUCLIDES,
C     INDICATOR BACTERIA), AND UN-IONIZED AMMONIA CONSTITUENTS.
C     THIS PROGRAM INCLUDES OPTIONS FOR DESIGN : QRIVER,QEFFL,TEMP & PH.
C      PROGRAM DEVELOPED BY T. P. H. GOWDA, WATER RESOURCES BRANCH
C       DATE: JUNE 1980.
C  DECLARATION STATEMENTS
C
       DIMENSION TITLE(20),C(10,50),CUI(10,50),PH(4)
       DIMENSION ARAY1(400,10),ARAY2(400,4),ARAY3(400,4)
       REAL*8 X(10),XX(10),PBK,TBK,PXWC,TXWC,A3,RKS(10),QY(50),QRS,
      1THETA,BPWR,HPWR,UPWR,QRTO,QT,QRUP(6),QEFL(6),CTDP,PHDG,VOL(10),
      2B(10),H(10),U(10),BSUM(10),TOT(10),TMP(6),RQ(11),TEMPS,PAX1,PAX2,
      3RF(10),BS(10),HS(10),US(10),BETA(10),BW(10),HW(10),UW(10),RKT(10),
      4PHI(10),R,QCP,PQX,EY(10),XCRIT,PCRIT,XEK,XMZ,PMZ,TMZ,RBK,RCRT,REK,
      5RXS,RKAV,AWCP,RFWC,XWCP,RBT,RBKG,XSCE,XSCEA,PAX1,PAX2,PW1,PW2,QCR,
      6PHWC,DELQ,XL
       CALL ERRSET(208,999,-1)
C
C  INPUT DATA
   37 WRITE(6,2)
      READ(1,3) TITLE
      WRITE(6,4)
      READ(1,*) QRS,BPWR,HPWR,UPWR,TEMPS,NTR
      WRITE(6,6)
      DO 15 I=1,NTR
   15 READ(1,*) X(I),BS(I),HS(I),US(I)
      WRITE(6,8)
      READ(1,*) (BETA(I),I=1,NTR)
      WRITE(6,80)
      READ(1,*) MQ,(QRUP(J),J=1,MQ)
      WRITE(6,82)
      READ(1,*)MT,(TMP(L),L=1,MT)
      WRITE(6,86)
      READ(1,*) MF,(QEFL(L),L=1,MF)
   35 WRITE (6,52)
      READ(1,*) AMONIA
      IF(AMONIA.LE.0)GO TO 70
      WRITE(6,57)
      READ(1,*)MPH,(PH(JPH),JPH=1,MPH)
   70 CONTINUE
      WRITE(6,56)
      READ(1,*)QCP,CEFL,CBKG,CS,THETA,RBK,XWCP
      WRITE(6,55)
      READ(1,*) (RKS(I),I=1,NTR)
C  CALCULATE FLOW & TEMP'R SCALE-UP PARAMETERS
C     DELTA=0.0001;    ALOG(1/DELTA)=(4.0*2.3026) FOR CALCN. OF BGX
      NQR=11
      NW=6
      DO 10 L=1,NQR
   10 RQ(L)=(L-1)/10.0
      WRITE(6,24) TITLE
      IRUN=0
C     BEGIN COMPUTATIONS FOR THE INPUT OPTIONS.
      DO 20 JF=1,MF
      DO 20 JQ=1,MQ
      QT=QRUP(JQ)+QEFL(JF)
      DELQ=QT/(NQR-1)
      QRTO=QT/QRS
      CA=(CEFL*QEFL(JF)/QT)
      DO 20 JT=1,MT
      CTDP=THETA**(TMP(JT)-TEMPS)
      RBT=RBK*CTDP
      IF(AMONIA.LE.0)MPH=1
      DO 20 JPH=1,MPH
      IRUN=IRUN+1
      IR=IRUN
      IF(AMONIA.LE.0)PH(JPH)=7.0
      ARAY1(IR,1)=IRUN
      ARAY1(IR,2)=QEFL(JF)
      ARAY1(IR,3)=QRUP(JQ)
      ARAY1(IR,4)=TMP(JT)
      ARAY1(IR,5)=PH(JPH)
      WRITE(6,92)IRUN
C     CALCULATE AMMONIA IONIZATION PARAMETER
      IF(AMONIA.LE.0) GO TO 34
      PKA=0.09018+2729.92/(TMP(JT)+273.2)
      PF=PKA-PH(JPH)
      PCTU=1./(1.+10.**PF)
      CSTTL=CS/PCTU
```

A.5: LIST OF MIXAPPLN PROGRAM (CONTINUED)

```
   34 WRITE(6,88)QEFL(JF),QRUP(JQ),TMP(JT),PH(JPH),CEFL,CS
      IF(AMONIA.GE.1)WRITE(6,54)CSTTL
      WRITE(6,90) (RQ(L),L=1,NW)
      DO 12 I=1,NTR
      BSUM(I)=0.
      VOL(I)=0.0
   12 TOT(I)=0.
C     BEGIN COMPUTATIONS AT TRANSECT,I.
      DO 14 I=1,NTR
C   CALCULATE B,H,U FOR DESIGN FLOW=QT, FROM LEOPOLD-MADDOCK EQNS.
      B(I)=BS(I)*QRTO**BPWR
      H(I)=HS(I)*QRTO**HPWR
      U(I)=US(I)*QRTO**UPWR
C   CALCULATE WEIGHTED MEAN VALUES BW,HW,UW FROM OUTFALL TO TRANSECT(I)
      IF(I.GE.2) GO TO 60
      XX(1)=X(1)
      BW(1)=B(1)
      HW(1)=H(1)
      UW(1)=U(1)
      BSUM(1)=B(1)*XX(1)
      VOL(1)=XX(1)*B(1)*H(1)
      TOT(1)=XX(1)/U(1)
      GO TO 62
   60 I1=I-1
      XX(I)=X(I)-X(I1)
      BSUM(I)=BSUM(I1)+0.5*XX(I)*(B(I1)+B(I))
      VOL(I)=VOL(I1)+0.25*XX(I)*(B(I1)+B(I))*(H(I1)+H(I))
      TOT(I)=TOT(I1)+XX(I)/U(I)
      BW(I)=BSUM(I)/X(I)
      HW(I)=VOL(I)/(X(I)*BW(I))
      UW(I)=QT/(BW(I)*HW(I))
C   CALCULATE PRODUCT FUNCTION FOR DECAY, RF(I), & DISPERSION FACTOR
   62 RKT(I)=CTDP*RKS(I)
      PBK=RBT*TOT(I)
      CALL PDET(PBK,TBK)
      CBKX=CBKG*TBK
      A3=(RKT(I)*XX(I))/U(I)
      R=DEXP(-A3)
      IF(I.GE.2) GO TO 64
      RF(1)=R
      GO TO 66
   64 RF(I)=RF(I1)*R
   66 EY(I)=BETA(I)*B(I)*U(I)
      PHI(I)=BETA(I)*X(I)/BW(I)
      PHDG=4.0*PHI(I)
      CMAX=0.5*CA*RF(I)/DSQRT(3.1416*PHI(I))
C  LATERAL CONC. DISTR'N AT TRANSECT,I.
      DO 16 K=1,NW
      QY(K)=(K-1)*DELQ
      IF (QY(K).GT.QT) QY(K)=QT
      PAX1=(QY(K)-QCP)/QT
      PAX2=(QY(K)+QCP)/QT
      CALL SUMSRS(PAX1,PAX2,PHDG,SUMT)
      C(I,K)=CMAX*SUMT+CBKX
   16 CONTINUE
      WRITE(6,25) X(I),EY(I),(C(I,K),K=1,NW)
   14 CONTINUE
C  UN-IONIZED AMMONIA CONCENTRATION DISTRN.
      IF(AMONIA.LE.0)GO TO 19
      WRITE(6,26)
      DO 19 I=1,NTR
      DO 17 K=1,NW
   17 CUI(I,K)=C(I,K)*PCTU
      WRITE(6,25) X(I),EY(I),(CUI(I,K),K=1,NW)
   19 CONTINUE
C     END OF COMPUTATIONS AT TRANSECT,I.
      IF(AMONIA.LE.0)CSL=CS
      IF(AMONIA.GE.1)CSL=CSTTL
C     COMPUTE BACKGROUND AVG. CONC. AT D/S WPCP.
      RKAV=-DLOG(RF(NTR))/TOT(NTR)
      AWCP=RKAV*XWCP/UW(NTR)
      CALL PDET(AWCP,RFWC)
      CBA=CA*RFWC
   76 PXWC=RBT*XWCP/UW(NTR)
      CALL PDET(PXWC,TXWC)
      CBB=CBKG*TXWC
      CAWP=CBA+CBB
C     COMPUTE BANK CONC. AT D/S WPCP.
      PW1=0.
      PW2=0.
      PHWC=4.0*BETA(NTR)*XWCP/BW(NTR)
```

A.5: LIST OF MIXAPPLN PROGRAM (CONTINUED)

```
        CALL SUMSRS(PW1,PW2,PHWC,SUMWC)
        CWCP=CBA*SUMWC/DSQRT(3.1416*PHWC)+CBB
C   COMPUTATIONS FOR MIXING ZONE PARAMETERS
        XMZ=B(NTR)/BETA(NTR)
        PMZ=RKAV*XMZ/UW(NTR)
        CALL PDET(PMZ,TMZ)
        CMZ=CA*TMZ
C       CALCULATE XSCE
        IF(CWCP.GT.CSL) GOTO 220
        CALL PARSPR(C,CSL,X,NTR,KXS,XEST,CXS)
        RXS=-DLOG(RF(KXS))/TOT(KXS)
        CALL SPREAD(CSL,CA,CBKG,RBT,CXS,XEST,XSCE,KXS,BW,RXS,UW,BETA,ITRN)
        GO TO 222
   220  XSCE=-999.0
        ITRN=0
   222  WRITE(6,50)XSCE,ITRN,XMZ,CMZ,XWCP,CWCP,CAWP
        ARAY1(IR,6)=CAWP
        ARAY1(IR,7)=XSCE
C   CRITICAL POINT RESULTS FOR QRATIO=0.2 , 0.3 & 0.4
        WRITE(6,94)
        DO 20 K=3,5
        KK=K-2
        CCRIT=-1.0E+10
C   SEARCH FOR TRANSECT NEAR WHICH CRIT. CONC. OCCURS TO FIND MOVING
C       AVG. VALUES FOR CRITICAL POINT COMPUTATIONS
        DO 18 I=1,NTR
        IF(CCRIT.GE.C(I,K)) GO TO 18
        ICR=I
        CCRIT=C(I,K)
    18  CONTINUE
        XCRIT=X(ICR)
        PCRIT=PHI(ICR)
        RCRT=-DLOG(RF(ICR))/TOT(ICR)
C     CALCULATE XL,CL & CEA BY CRIT. POINT METHOD USING MOVING AVG. VALUES
    40  QCR=4.0*RCRT*XCRIT/(PCRIT*UW(ICR))*(QY(K)/QT)**2
        XL=0.25*UW(ICR)*(-1.0+DSQRT(1+QCR))/RCRT
        XLDIF=DABS(XL-XCRIT)
        XPCT=100*XLDIF/XL
        IF(XPCT.LE.5.) GO TO 42
        XCRIT=XL
        PCRIT=BETA(ICR)*XL/BW(ICR)
        GO TO 40
    42  PQX=(RCRT*XL/UW(ICR))+(0.25/PCRIT*(QY(K)/QT)**2)
        CL=(CA/DSQRT(3.1416*PCRIT))*DEXP(-PQX)+CBKG*DEXP(-RBT*XL/UW(ICR))
        CEA=CEFL*CSL/CL
C       COMPUTE XSCEA
        IF(CWCP.GT.CL)GO TO 114
        CALL PARSPR(C,CL,X,NTR,IEK,XEK,CXK)
        REK=-DLOG(RF(IEK))/TOT(IEK)
        CALL SPREAD(CL,CA,CBKG,RBT,CXK,XEK,XSCEA,IEK,BW,REK,UW,BETA,ITRA)
        GO TO 116
   114  XSCEA=-999.0
        ITRA=0
   116  WRITE(6,96)RQ(K),XL,CL,CEA,XSCEA,ITRA
C       STORE OUTPUT IN ARRAYS
        GO TO (45,46,47),KK
    45  ARAY1(IR,8)=RQ(3)
        ARAY1(IR,9)=CEA
        ARAY1(IR,10)=XSCEA
        GO TO 20
    46  ARAY2(IR,1)=RQ(4)
        ARAY2(IR,2)=CEA
        ARAY2(IR,3)=XSCEA
        GO TO 20
    47  ARAY3(IR,1)=RQ(5)
        ARAY3(IR,2)=CEA
        ARAY3(IR,3)=XSCEA
    20  CONTINUE
C       END OF COMPUTATIONS FOR THE INPUT OPTIONS
C       PRINT SUMMARIZED RESULTS
        WRITE(6,78)TITLE
        WRITE(6,100)
        DO 48 N=1,IR
        WRITE(6,102)(ARAY1(N,J),J=1,10)
        WRITE(6,104)(ARAY2(N,J),J=1,3)
    48  WRITE(6,104)(ARAY3(N,J),J=1,3)
C   CHANGE PARAMETERS
        WRITE(6,28)
        READ(1,*) NCHNG
        GO TO(35,37,999),NCHNG
C   FORMAT STATEMENTS
```

A.5: LIST OF MIXAPPLN PROGRAM (CONTINUED)

```
    2  FORMAT(/' ENTER TITLE OF STUDY')
    3  FORMAT(20A4)
    4  FORMAT(' ENTER QRS,BPWR,HPWR,UPWR,TEMPS,NTR')
    6  FORMAT(' ENTER NTR VALUES OF X,BS,HS,US')
    8  FORMAT(' ENTER NTR VALUES OF BETA')
   22  FORMAT(2A4)
   24  FORMAT(1H1//6X,'PREDICTIONS OF RUNS FOR MANAGEMENT OPTIONS'/20A4)
   25  FORMAT(2X,F8.1,1X,F7.4,11F9.3)
   26  FORMAT(2X,'TOXIC AMMONIA:')
   28  FORMAT(/'  TO CHANGE PARAMETERS ENTER THE NUMBER STATED BELOW'/
      *'    AMMONIA: 1;     TITLE: 2;     END: 3.')
   50  FORMAT(/5X,'XS (WITH CE)=',F8.1,3X,'NO. OF ITERATIONS=',I2,/5X,
      *'MIXING ZONE LENGTH=',F9.1,5X,'CONC=',F8.2/5X,'DIST. TO D/S WPCP='
      *,F8.1,5X,'SHORE CONC. AT D/S WPCP=',F6.2/8X,'AVG. CONC.AT D/S WPCP
      *= ',F6.2)
   52  FORMAT('    UN-IONIZED AMMONIA: ENTER 1 FOR YES; 0 FOR NO')
   54  FORMAT(5X,'  CRITERION FOR TOTAL AMMONIA, CS=',F6.3)
   55  FORMAT(2X,' ENTER NTR VALUES OF RKS')
   56  FORMAT('    ENTER QCP,CEFL,CBKG,CS,THETA,RKB,XWCP')
   57  FORMAT('    ENTER MPH AND PH VALUES')
   78  FORMAT(1H1//6X,'SUMMARY OF RUNS FOR MANAGEMENT OPTIONS'/20A4)
   80  FORMAT('  ENTER MQ & QRUP VALUES')
   82  FORMAT('  ENTER MT & TMP VALUES')
   86  FORMAT('  ENTER MF & QEFL VALUES')
   88  FORMAT(2X,'QEFL=',F7.3,2X,'QRUP=',F7.3,2X,'TEMPR=',F4.1,2X,'PH=',
      *F4.1,2X,'CEFL=',F7.2,2X'CS=',F5.2)
   90  FORMAT(/6X,'X',6X,'EY',7X,6(F3.1,6X),/)
   92  FORMAT(/'    * * RUN NO.: ',I4)
   94  FORMAT(/9X,'     CRITICAL POINT METHOD RESULTS '/9X,'QY/QT',6X,
      *'XL',8X,'CL',9X,'CEA',6X,'XSCEA',6X,'ITRN')
   96  FORMAT(9X,F4.2,4X,F6.1,3X,F7.3,4X,F8.2,3X,F8.2,4X,I4)
  100  FORMAT(//1X,'RUN#',3X,'QEFL',5X,'QPUP',3X,'TEMP',4X,'PH',4X,
      1'CAWP',5X,'XSCE',4X,'QY/QT',4X,'CEA',5X,'XSCEA'/)
  102  FORMAT(1X,F4.0,2X,F6.3,2X,F6.2,2(2X,F5.1),2X,F6.3,
      *2X,F8.1,2X,F4.2,2X,F7.2,2X,F8.1)
  104  FORMAT(55X,F4.2,2X,F7.2,2X,F8.1)
  999  STOP
       END
C
C   SUBROUTINE FOR SUMMATION OF EXPONENTIAL SERIES TERMS.
       SUBROUTINE SUMSRS(QAX1,QAX2,PFDR,SUM)
       REAL*8 QAX1,QAX2,PFDR,BGX,SBG,P1,P2,P3,P4,T1,T2,T3,T4
       BGX=2.3026*PFDR
       SBG=DSQRT(BGX)
C  DETERMINE NO. OF IMAGES REQUIRED
       AN1=(0.5*QAX1-SBG)-0.5
       AN2=(0.5*QAX1+SBG)+0.5
       AN3=-AN2
       AN4=-AN1
       NM1=IFIX(AN1)
       NM2=IFIX(AN2)
       NM3=IFIX(AN3)
       NM4=IFIX(AN4)
       NN1=1+NM2+IABS(NM1)
       NN2=1+IABS(NM3)+IABS(NM4)
       IF(NN1.GE.NN2)NN=NN1+1
       IF(NN1.LT.NN2)NN=NN2+1
C   COMPUTE SUM OF EXPONENTIAL SERIES TERMS
       SUM=0.
       DO 32 J=1,NN
       N=J-1
       P1=(QAX1-2.*N)**2/PFDR
       P2=(QAX2+2.*N)**2/PFDR
       CALL PDET(P1,T1)
       CALL PDET(P2,T2)
       IF(N.LE.0) GO TO 30
       P3=(QAX1+2.*N)**2/PFDR
       P4=(QAX2-2.*N)**2/PFDR
       CALL PDET(P3,T3)
       CALL PDET(P4,T4)
       GO TO 32
   30  T3=0.
       T4=0.
   32  SUM=SUM+T1+T2+T3+T4
       RETURN
       END
C
C  COMPUTE EXPONENTIAL TERMS: SET (P.LE.40.0) TO AVOID ERROR 208, SO
C    THAT EXP(-P)=4.3E-18.
       SUBROUTINE PDET(P,T)
       REAL*8 P,T
```

A.5: LIST OF MIXAPPLN PROGRAM (CONTINUED)

```
      IF(P.GE.40.)GO TO 10
      T=DEXP(-P)
      GO TO 12
 10   T=0.0
 12   CONTINUE
      RETURN
      END
C
C  COMPUTE PARAMETERS IN THE SUBROUTINE 'SPREAD'.
      SUBROUTINE PARSPR(C,CL,X,NTR,IEK,XEK,CXK)
      DIMENSION C(1,1)
      REAL*8  X(1),XEK
      IF(C(NTR,1).GE.CL) GO TO 75
      DO 72 I=2,NTR
      II=I-1
      IF(C(1,1).LE.CL)GO TO 73
      IF(C(II,1).GT.CL.AND.C(I,1).LE.CL) GO TO 73
 72   CONTINUE
 73   IEK=II
      GO TO 79
 75   IEK=NTR
 79   XEK=X(IEK)
      CXK=C(IEK,1)
      RETURN
      END
C
      SUBROUTINE SPREAD(CST,CAV,CBG,RB,CXY,XEY,XST,M,BI,RS,US,BTA,IT)
C   COMPUTATION OF MAX. LONGL. SPREAD,XS, ALONG OUTFALL BANK WHERE
C      (C(XS,0)-CS)=5 PERCENT(ABSOLUTE).
      REAL*8 BI(1),BTA(1),RS,US(1),PS,TS,PHB,RFS,RB,XS,PX1,PX2,TG,PG
      CXX=CXY
      XS=XEY
      IT=0
 18   IT=IT+1
      DIFF=(CXX-CST)
      RCAB=ABS(DIFF/CST)
      PRCNT=100.*RCAB
      IF(PRCNT.LE.5.)GO TO 26
      IF(IT.GT.30) GO TO 27
      FRX=RCAB/(1.0+PS)
      IF(DIFF.LE.0.0) XS=XS*(1-FRX)
      IF(DIFF.GT.0.0) XS=XS*(1+FRX)
      PHB=4.0*BTA(M)*XS/BI(M)
      PS=RS*XS/US(M)
      CALL PDET(PS,TS)
      CMM=CAV*TS/DSQRT(3.1416*PHB)
      PX1=0.
      PX2=0.
      PG=RB*XS/US(M)
      CALL PDET(PG,TG)
      CALL SUMSRS(PX1,PX2,PHB,SUMP)
      CXX=CMM*SUMP+CBG*TG
      GO TO 18
 27   XS=-888.0
 26   XST=XS
      RETURN
      END
```

A.6: TYPICAL INPUT/OUTPUT OF MIXAPPLN PROGRAM

INPUT:

```
ENTER TITLE OF STUDY
ENTER QRS,BPWR.HPWR.UPWR.TEMPS.NTR
ENTER NTR VALUES OF X.BS.HS.US
ENTER NTR VALUES OF BETA
 ENTER MQ & QRUP VALUES
 ENTER MT & TMP VALUES
 ENTER MF & QEFL VALUES
. UN-IONIZED AMMONIA: ENTER 1 FOR YES; 0 FOR NO
  ENTER MPH AND PH VALUES
  ENTER QCP.CEFL,CBKG.CS.THETA.RKB.XWCP
  ENTER NTR VALUES OF RKS

TO CHANGE PARAMETERS ENTER THE NUMBER STATED BELOW
 AMMONIA: 1;    TITLE: 2;    END: 3.


    AMMONIA IN MIXING ZONE-GRAND RIVER BELOW WATERLOO WPCP *
12.54 .05 .5 .45 23 5
122 54.86 .62 .37
427 86.87 .62 .63
1082 44.2 .51 .55
2240 59.44 .49 .43
3398 50.29 .69 .37
.0025 .0020 .0013 .0008 .0010
1 1.132
2 20 25
1 0.307
1
1 8.3
0 12.5 0 0.02 1.106 0 20487
.0000231 .0000231 .0000231 .0000231 .0000231
3
```

OUTPUT OF MIXAPPLN PROGRAM

PREDICTIONS OF RUNS FOR MANAGEMENT OPTIONS
AMMONIA IN MIXING ZONE-GRAND RIVER BELOW WATERLOO WPCP *

* * RUN NO.: 1
QEFL= 0.307 QRUP= 1.132 TEMPR=20.0 PH= 8.3 CEFL= 12.50 CS= 0.02
CRITERION FOR TOTAL AMMONIA, CS= 0.272

X	EY	0.0	0.1	0.2	0.3	0.4	0.5
122.0	0.0172	18.832	12.579	3.749	0.498	0.030	0.001
427.0	0.0371	12.104	10.169	6.031	2.525	0.746	0.156
1082.0	0.0107	8.906	8.018	5.851	3.461	1.660	0.645
2240.0	0.0069	6.590	6.124	4.914	3.405	2.038	1.053
3398.0	0.0063	4.108	3.956	3.531	2.923	2.243	1.596
TOXIC AMMONIA:							
122.0	0.0172	1.383	0.924	0.275	0.037	0.002	0.000
427.0	0.0371	0.889	0.747	0.443	0.185	0.055	0.011
1082.0	0.0107	0.654	0.589	0.430	0.254	0.122	0.047
2240.0	0.0069	0.484	0.450	0.361	0.250	0.150	0.077
3398.0	0.0063	0.302	0.291	0.259	0.215	0.165	0.117

XS (WITH CE)= 20259.4 NO. OF ITERATIONS= 8
MIXING ZONE LENGTH= 45130.5 CONC= 0.01
DIST. TO D/S WPCP= 20487.0 SHORE CONC. AT D/S WPCP= 0.26
AVG. CONC.AT D/S WPCP= 0.25

CRITICAL POINT METHOD RESULTS

QY/QT	XL	CL	CEA	XSCEA	ITRN
0.20	516.0	5.946	0.57	2312.38	2
0.30	1455.0	3.400	1.00	3956.84	2
0.40	2575.7	2.243	1.52	5735.51	3

* * RUN NO.: 2
QEFL= 0.307 QRUP= 1.132 TEMPR=25.0 PH= 8.3 CEFL= 12.50 CS= 0.02
CRITERION FOR TOTAL AMMONIA, CS= 0.196

X	EY	0.0	0.1	0.2	0.3	0.4	0.5
122.0	0.0172	18.649	12.457	3.712	0.494	0.029	0.001
427.0	0.0371	11.815	9.927	5.887	2.465	0.728	0.152
1082.0	0.0107	8.392	7.556	5.514	3.262	1.564	0.608
2240.0	0.0069	5.734	5.328	4.276	2.963	1.773	0.916
3398.0	0.0063	3.258	3.137	2.800	2.318	1.778	1.266
TOXIC AMMONIA:							
122.0	0.0172	1.902	1.270	0.379	0.050	0.003	0.000
427.0	0.0371	1.205	1.012	0.600	0.251	0.074	0.015
1082.0	0.0107	0.856	0.770	0.562	0.333	0.159	0.062
2240.0	0.0069	0.585	0.543	0.436	0.302	0.181	0.093
3398.0	0.0063	0.332	0.320	0.286	0.236	0.181	0.129

XS (WITH CE)= 14413.0 NO. OF ITERATIONS= 9
MIXING ZONE LENGTH= 45130.5 CONC= 0.00
DIST. TO D/S WPCP= 20487.0 SHORE CONC. AT D/S WPCP= 0.05
AVG. CONC.AT D/S WPCP= 0.05

CRITICAL POINT METHOD RESULTS

QY/QT	XL	CL	CEA	XSCEA	ITRN
0.20	481.3	5.665	0.43	2240.00	1
0.30	1284.3	3.002	0.82	3474.97	2
0.40	2221.0	1.872	1.31	4907.67	2

SUMMARY OF RUNS FOR MANAGEMENT OPTIONS
AMMONIA IN MIXING ZONE-GRAND RIVER BELOW WATERLOO WPCP *

RUN#	QEFL	QPUP	TEMP	PH	CAWP	XSCE	QY/QT	CEA	XSCEA
1.	0.307	1.13	20.0	8.3	0.248	20259.4	0.20	0.57	2312.4
							0.30	1.00	3956.8
							0.40	1.52	5735.5
2.	0.307	1.13	25.0	8.3	0.052	14413.0	0.20	0.43	2240.0
							0.30	0.82	3475.0
							0.40	1.31	4907.7

TO CHANGE PARAMETERS ENTER THE NUMBER STATED BELOW
AMMONIA: 1; TITLE: 2; END: 3.

APPENDIX B

DOCUMENTATION OF MIXANDAT PROGRAM

B.1: DESCRIPTION OF VARIABLE NAMES

TITLE:	title of study
NTR:	number of transects
OUTBNK:	outfall bank of river (left or right looking upstream)
QEFL:	effluent flow rate
FEMT:	conversion factor for measurement units
Fl & F2:	constants in the Resistance Equation used for velocity synthesis
TRNSCT:	name of transect
X:	distance below outfall
NYZ:	number of lateral data points at given transect
Q, QRIVER:	river discharge at given transect
MVEL:	identifier of transect at which velocities are measured
REFLD:	river bank at given transect w.r.t. which data was collected during field study
YL:	lateral distance at a given transect w.r.t. REFLD
ZL:	local depth at distance YL
UVL:	depth averaged local velocity at distance YL
CPARM:	name of tracer or pollutant parameter
CBKG:	background concentration of CPARM in river water
CEFL:	effluent concentration of CPARM
CONL:	depth-averaged local concentration at distance YL
Y:	lateral distance at a given transect w.r.t. the outfall bank, OUTBANK
Z:	local depth at distance, Y
VEL:	local velocity at distance, Y
CONC:	net concentration at distance, y (obtained by subtracting CBKG from measured concentration value).
SUMA:	partial cumulative area between OUTBNK and lateral distance, Y

SUMQ: partial cumulative discharge, q, between OUTBNK and lateral distance, Y

SUMF: partial cumulative mass flux between OUTBNK and lateral distance, Y

Y/B: lateral distance, Y, divided by channel width, B, at a given transect

QY/QT: SUMQ divided by total river discharge

CAVG: completely mixed instream average concentration just below outfall

CATRN: cross-sectional average concentration at a given transect, c_{atr}, calculated from (total net mass flux/river discharge)

C/CAVG: ratio between CONC and CAVG

C/CATRN: ratio between CONC AND CATRN

VCMAX: variance, ^2_y, from maximum concentration method

VCN: variance, ^2_y, from second moment of c-y distribution curve

VUF: variance from second moment of unit flux distribution curve

VPQ: variance, ^2_q, from maximum concentration method

VCQ: variance, ^2_q, from second moment of c-q distribution curve

X/B: ratio between downstream distance, X, and channel width, B

X/H: ratio between X and mean depth, H

VCN/BB: ratio (VCN/B^2)

VCN/HH: ratio (VCN/H^2)

VUF/BB: ratio (VUF/B^2)

VUF/HH: ratio (VUF/H^2)

VCQ/QQ: ratio (VCQ/Q^2)

B.2: INPUT DATA PREPARATION

The input data for the MIXANDAT program may be entered from cards or an auxiliary input device such as a disk file. The following details should be noted in preparing the input data as outlined in Table B1.

1. The variables OUTBNK and REFLD, which identify the river banks, should be input as LFT or RHT to denote the left and right banks, respectively. However, when both OUTBNK and REFLD refer to the same bank, it is sufficient to state the input for OUTBNK; and REFLD can be left blank. With these inputs, the lateral co-ordinates of various data points are expressed with respect to the outfall bank.

2. When the input data are in British units, the factor FMET is set equal to 0.3048 to convert the input data on distances, widths, depths, velocities and flow rates from the British units to the metric units. If input data are already in metric units, then FMET can be left blank or set equal to 1.0. The default value is taken to be 1.0 when FMET is left blank. Note that all the input parameters should be in either British units or metric units.

3. If the coefficients F1 and F2 are not known, they can be left blank; then default values are taken to be 1.0 and 0.67, respectively.

4. MVEL must be specified to be 99 for transects at which velocity distribution data are available, and left blank otherwise.

5. The maximum number of some input parameters are as follows:

   No. of transects, NTR = 8
   No. of lateral data points at a transect, NYZ = 60;
   No. of tracer plus pollutant parameters at a transect, JP = 5.

6. The first two cards (i.e. card #1 and 2) contain input data of a general nature (i.e. not dependent on transect).

7. The number of cards containing input data at a given transect are dependent on: (i) NYZ; (ii) presence or absence of velocity data; and (iii) JP. Thus, in Table B1, the card numbers are shown as n1, n2, etc.

8. The end of input data at each transect is identified by specifying NOCONC for CPARM and -999.0 for CBKG, as in Table B1.

TABLE B1: INPUT DATA FORMAT FOR MIXANDAT PROGRAM

VARIABLE	CARD NO. & (FORMAT)	COLUMNS	DESCRIPTION	REMARKS
TITLE	1 (20A4)	1-80	study title, survey date, etc.	cards 1 & 2 include general data.
NTR	2 (I2)	1-2	number of transects	
OUTBNK	2 (A3)	6-8	outfall bank of river, LFT or RHT	
QEFL	2 (F10.0)	11-20	effluent discharge	
FMET	2 (F10.0)	21-30	conversion factor	default value is 1.0
F1	2 (F10.0)	31-40	constant in resistance equation	default value is 1.0
F2	2 (F10.0)	41-50	exponent in resistance equation	default value is 0.67
TRNSCT	3 (5A4)	1-20	transect name	beginning of input data at 1st transect starts on Card #3.
X(I)	3 (F10.0)	21-30	distance to transect below outfall	
NYZ	3 (I5)	31-35	No. of lateral data points	
Q(I)	3 (F10.0)	36-45	river discharge at the transect	
MVEL	3 (I5)	46-50	identifier for velocity distribution	
REFLD	3 (A3)	56-58	reference bank for data, LFT or RHT	default value is the same as that of OUTBNK.
YL(I,J)	4 to n1 (10F8.0)	multiples of 8 (1-8, 9-16, etc.)	NYZ values of lateral distances	10 values per card.
ZL(I,J)	n2 to n3 (10F8.0)	multiples of 8	NYZ values of depth	10 values per card.

VARIABLE	CARD NO. & (FORMAT)	COLUMNS	DESCRIPTION	REMARKS
UVL(I,J)	n4 to n5 (10F8.0)	multiples of 8	NYZ values of velocities	optional card(s); 10 values per card.
CPARM	n6 (5A4)	1 to 20	name of parameter (eg. chloride ion)	concentration data for JP = 1 at 1st transect on the group of cards n6 to n8.
CBKG	n6 (F10.0)	21 to 30	background concentration	
CEFL	n6 (F10.0)	31 to 40	effluent concentration	
CONL(I,J)	n7 to n8 (10F8.0)	multiples of 8	NYZ values of concentrations	
CPARM	n9	1 to 20	name of parameter	The card group containing inputs CPARM to CONL may be repeated upto JP = 5.
.	.	.	.	
.	.	.	.	
.	.	.	.	
.	.	.	.	
CPARM	nj (5A4)	1 to 20	end of input data at 1st transect	specify NOCONC
CBKG	nj (F10.0)	21 to 30	"	specify -999.0
TRNSCT	nk (5A4)	1 to 20	transect name	beginning of input data for 2nd transect, and so on.

B.3: LIST OF MIXANDAT PROGRAM

```
C     DATA ANALYSIS PROGRAM 'MIXANDAT':
C       PROGRAMMER : T.P.H.GOWDA
C
        REAL LEFT
        DIMENSION CPARM(5),X(8),Y(8,60),Z(8,60),VEL(8,60),CONC(8,60)
        DIMENSION YL(8,60),ZL(8,60),CONL(8,60),UVL(8,60),VUF(5),VCN(5)
        DIMENSION SUMA(60),DVEL(60),DELQ(60),SUMQ(60),DCONC(60),SBF(5)
        DIMENSION FLUX(60),DELA(60),SUMF(60),TITLE(20),TRNSCT(5),SHF(5)
        DIMENSION Q(8),U(8),YW(60),UNIF(60),VCMX(5),SBS(5),SHS(5)
        DIMENSION CMX(5),VCQ(5),SBQ(5),SHQ(5),SQQ(5),VPQ(5)
        DATA LEFT/'LFT'/,BLANK/'   '/,RIGHT/'RHT'/
C        DATA INPUT ***
        READ(1,10)TITLE
        READ(1,12)NTR,OUTBNK,QEFL,FMET,F1,F2
        IF(FMET.LE.0.0) FMET=1.0
        IF(F1.LE.0.0) F1=1.0
        IF(F2.LE.0.0) F2=0.67
        QEFL=QEFL*FMET**3
        WRITE(6,30)TITLE
        DO 100 I=1,NTR
        READ(1,16)TRNSCT,X(I),NYZ,Q(I),MVEL,REFLD
        READ(1,18)(YL(I,J),J=1,NYZ)
        READ(1,18)(ZL(I,J),J=1,NYZ)
        IF(REFLD.NE.BLANK) GO TO 38
        IF(OUTBNK.EQ.LEFT) REFLD=LEFT
        IF(OUTBNK.EQ.RIGHT) REFLD=RIGHT
   38   CONTINUE
        IF(MVEL.EQ.99) READ(1,18)(UVL(I,J),J=1,NYZ)
        X(I)=X(I)*FMET
        Q(I)=Q(I)*FMET**3
        KYZ=NYZ-1
        JP=0
    99  READ(1,20) CPARM,CBKG,CEFL
        IF(CBKG.EQ.-999.0.OR.JP.GE.5)GO TO 150
        JP=JP+1
        READ(1,18)(CONL(I,J),J=1,NYZ)
C       EXPRESS DATA W.R.T  ORIGIN AT OUTFALL BANK SIDE OF TRANSECT
        CMX(JP)  =-1.0E+10
        DO 130 J=1,NYZ
        IF(OUTBNK.EQ.LEFT.AND.REFLD.EQ.LEFT) GO TO 40
        IF(OUTBNK.EQ.RIGHT.AND.REFLD.EQ.RIGHT) GO TO 40
        K=NYZ-J+1
        GO TO 42
   40   K=J
   42   CONTINUE
        IF(JP.GE.2) GO TO 120
        Y(I,K)=YL(I,J)*FMET
        YW(K)=Y(I,K)
        Z(I,K)=ZL(I,J)*FMET
        IF(MVEL.EQ.99) VEL(I,K)=UVL(I,J)*FMET
  120   CONC(I,K)=CONL(I,J)-CBKG
        IF(CONC(I,K).LT.0.0) CONC(I,K)=0.0
C     FIND PEAK CONC. & ITS POSITION AT TRANSECT.
        IF(CMX(JP).GE.CONC(I,K)) GO TO 130
        KP=K
        CMX(JP)=CONC(I,K)
  130   CONTINUE
C
        BKFX=CBKG*(Q(I)-QEFL)
        EFLX=CEFL*QEFL
        TFLX=BKFX+EFLX
        IF(I.GT.1)WRITE(6,198)
        WRITE(6,200)TRNSCT,X(I),JP,CPARM,Q(I),CBKG,QEFL,CEFL,BKFX,EFLX,
       *TFLX
        IF(MVEL.NE.99)WRITE(6,210)
        IF(MVEL.EQ.99)WRITE(6,212)
        WRITE(6,202)
        IF(JP.GE.2) GO TO 204
C     COMPUTE AREA & DISCHARGE.
        SUMA(1)=0.
        SUMQ(1)=0.
        DO 22 J=1,KYZ
        JJ=J+1
        DELA(J)=0.5*(Y(I,JJ)-Y(I,J))*(Z(I,JJ)+Z(I,J))
        SUMA(JJ)=SUMA(J)+DELA(J)
        IF(MVEL.NE.99) GO TO 22
        DVEL(J)=0.5*(VEL(I,JJ)+VEL(I,J))
        DELQ(J)=DELA(J)*DVEL(J)
        SUMQ(JJ)=SUMQ(J)+DELQ(J)
```

```
   22 CONTINUE
      ZAV=SUMA(NYZ)/Y(I,NYZ)
      IF(MVEL.NE.99) U(I)=Q(I)/SUMA(NYZ)
      IF(MVEL.EQ.99)U(I)=SUMQ(NYZ)/SUMA(NYZ)
C   VELOCITY SIMULATION USING RESISTANCE(EG. MANNING'S) EQN.
      IF(MVEL.EQ.99) GO TO 106
      DO 104 J=1,NYZ
  104 VEL(I,J)=F1*U(I)*(Z(I,J)/ZAV)**F2
C  ESTIMATE DISCHARGE FROM SIMULATED VEL. DISTR'N.
      SUMQ(1)=0.0
      DO 108 J=1,KYZ
      JJ=J+1
      DVEL(J)=0.5*(VEL(I,JJ)+VEL(I,J))
      DELQ(J)=DELA(J)*DVEL(J)
  108 SUMQ(JJ)=SUMQ(J)+DELQ(J)
  106 CONTINUE
C   VELOCITY CORRECTION TO CONFORM WITH SUMQ(NYZ)=Q(I).
      DO 109 J=1,NYZ
  109 VEL(I,J)=VEL(I,J)*Q(I)/SUMQ(NYZ)
      SUMQ(1)=0.0
      DO 110 J=1,KYZ
      JJ=J+1
      DVEL(J)=0.5*(VEL(I,JJ)+VEL(I,J))
      DELQ(J)=DELA(J)*DVEL(J)
  110 SUMQ(JJ)=SUMQ(J)+DELQ(J)
      U(I)=SUMQ(NYZ)/SUMA(NYZ)
C  SHAPE-VELOCITY FACTOR.
      CALL CADIS(Z,VEL,ZAV,U,Q,DELQ,I,NYZ,SHAPE)
C   COMPUTE FLUX OF TRACER OR POLLUTANT.
  204 SUMF(1)=0.
      ARCY=0.
      DO 132 J=1,KYZ
      JJ=J+1
      IF(J.GT.KP.AND.CONC(I,J).LE.0.0)CONC(I,JJ)=0.0
      DCONC(J)=0.5*(CONC(I,JJ)+CONC(I,J))
      DCY=DCONC(J)*(Y(I,JJ)-Y(I,J))
      ARCY=ARCY+DCY
      FLUX(J)=DCONC(J)*DELQ(J)
      SUMF(JJ)=SUMF(J)+FLUX(J)
      UNIF(J)=FLUX(J)/(Y(I,JJ)-Y(I,J))
  132 CONTINUE
C  COMPUTE AVG. CONC'NS IN RIVER AT OUTFALL & TRANSECT.
      CAVG=CEFL*QEFL/Q(I)
      CATRN=SUMF(NYZ)/SUMQ(NYZ)
C  PRINT OUTPUT MATRIX OF Y,Z,VEL,SUMQ & SUMF
      DO 134 J=1,NYZ
      RYB=Y(I,J)/Y(I,NYZ)
      RQ=SUMQ(J)/SUMQ(NYZ)
      RC=CONC(I,J)/CAVG
      RCTRN=CONC(I,J)/CATRN
  134 WRITE(6,220)Y(I,J),Z(I,J),VEL(I,J),CONC(I,J),SUMA(J),
     *SUMQ(J),SUMF(J),RYB,RQ,RC,RCTRN
      FXTR=BKFX+SUMF(NYZ)
      WRITE(6,224)CAVG,CATRN,FXTR
      IF(JP.LE.1)WRITE(6,312) ZAV,U(I),SHAPE
C      VARIANCE COMPUTATION ***
C   VARIANCE FROM PEAK CONC'N.
      RLCN=CMX(JP)/ARCY
      VCMX(JP) =1.0/(6.2836*RLCN*RLCN)
C  VARIANCE FROM 2ND MOMENT OF C-Y DISTR'N.
      CALL VARANC(DCONC,NYZ,YW,VCN,ARCY,I,JP,KP,ZAV,SBS,SHS)
C  VARIANCE FROM 2ND MOMENT OF UNIT FLUX DISTR'N.
      FLXPK=-1.0E+10
      DO 142 J=1,KYZ
      IF(FLXPK.GE.FLUX(J)) GO TO 142
      KF=J
  142 CONTINUE
      ARUF=SUMF(NYZ)
      CALL VARANC(UNIF,NYZ,YW,VUF,ARUF,I,JP,KF,ZAV,SBF,SHF)
C  VARIANCE OF C-Q DISTRIBUTIONS
      CALL VARANC(DCONC,KYZ,SUMQ,VCQ,ARUF,I,JP,KF,ZAV,SBQ,SHQ)
      SQQ(JP)=VCQ(JP)/SUMQ(NYZ)**2
      RLCQ=CMX(JP)/ARUF
      VPQ(JP)=1.0/(6.2836*RLCQ*RLCQ)
      GO TO 99
  150 CONTINUE
      WRITE(6,230)TRNSCT
      DO 102 N=1,JP
      WRITE(6,234) N,VCMX(N),VCN(N),VUF(N),VCQ(N),VPQ(N)
```

```
  102 CONTINUE
      RXH=X(I)/ZAV
      RXB=X(I)/Y(I,NYZ)
      WRITE(6,240)RXB,RXH
      DO 100 N=1,JP
      WRITE(6,242)N,SBS(N),SHS(N),SBF(N),SHF(N),SQQ(N)
  100 CONTINUE
C   FORMAT STATEMENTS  * * * *
   10 FORMAT(20A4)
   12 FORMAT(I2,3X,A3,2X,4F10.0)
   16 FORMAT(5A4,2(F10.0,I5),5X,A3)
   18 FORMAT(10F8.0)
   20 FORMAT(5A4,2F10.0)
   30 FORMAT(1H1/5X,20A4/)
  198 FORMAT(1H1,/)
  200 FORMAT(/5X,5A4,2X,F8.1,' METERS FROM OUTFALL'/10X,'PARAMETER',
     11X,I1,':  ',5A4/5X,'QRIVER=',F9.3,5X,'BACKGROUND CONC.=',F8.3/5X,
     2'QEFL  =',F9.3,5X,'EFFLUENT CONCN.=',F9.3/5X,'UPSTREAM FLUX= ',
     3F8.2,3X,'EFFLUENT FLUX= ',F8.2,3X,'TOTAL FLUX = ',F9.2)
  202 FORMAT(5X,'Y',6X,'Z',5X,'VEL',5X,'CONC',6X,'SUMA',5X,'SUMQ',
     *5X,'SUMF',4X,'Y/B',4X,'QY/QT',3X,'C/CAVG',3X,'C/CATRN'/)
  210 FORMAT(10X,'VELOCITIES SIMULATED FROM RESISTANCE EQN.'/)
  212 FORMAT(10X,'MEASURED VELOCITIES CORRECTED TO GET Q=SUMQ(NYZ)'/)
  220 FORMAT(1X,F7.2,1X,F6.2,2X,F5.2,2X,3(F7.2,2X),F8.2,4(2X,F6.3))
  224 FORMAT( /5X,'AVG. CONC. JUST BELOW OUTFALL, CAVG=',F8.3/5X,'AVG. C
     1ONC. AT THE TRANSECT, CATRN  =',F8.3,5X,'TOTAL FLUX AT TRANSECT= '
     2,F9.2)
  230 FORMAT(/5X,5A4,':VARIANCE FROM DIFFERENT METHODS:'/4X,'PARAMETER'
     *,4X,'VCMAX',7X,'VCN',8X,'VUF',8X,'VCQ',8X,'VPQ'/)
  234 FORMAT( 7X,I2,3X,5(F10.2,1X))
  240 FORMAT(///5X,'NONDIMENSIONAL VARIANCE',5X,'X/B=',F8.2,5X,'X/H=',
     1F9.1//2X,'PARAMETER',4X,'VCN/BB',5X,'VCN/HH',5X,'VUF/BB',5X,
     2'VUF/HH',5X,'VCQ/QQ'/)
  242 FORMAT(5X,I2,6X,2(F8.4,2X,F9.2,3X),F9.4)
  312 FORMAT(5X,'MEAN DEPTH=',F6.3,5X,'MEAN VELOCITY=',F6.3,5X,'SHAPE-VE
     *LOCITY FACTOR=',F6.3)
 9999 STOP
      END
C
C  COMPUTATION OF SHAPE-VELOCITY FACTOR.
      SUBROUTINE CADIS(H,V,HA,VA,QR,DLQ,L,N,SHP)
      DIMENSION H(8,60),V(8,60),QR(8),DLQ(60),VA(8)
      SUMD=0.
      NS=N-1
      DO 12 J=1,NS
      JJ=J+1
      HR=((H(L,JJ)+H(L,J))/HA)**2
      VR=((V(L,JJ)+V(L,J))/VA(L))
      BHUY=DLQ(J)*HR*VR
   12 SUMD=SUMD+BHUY
      SHP=SUMD/(8.0*QR(L))
      RETURN
      END
C
C  SUBROUTINE TO COMPUTE VARIANCE VALUES FOR BANK OUTFALL CASE
      SUBROUTINE VARANC(P,NY,R,V,SUMD,IC,J,KI,Z,SB,SH)
      DIMENSION P(60),R(60),V(5),SB(5),SH(5)
      SUMN=0.
      K=NY-1
      M=1
   10 L=M+1
      IF(M.GT.KI.AND.P(M).LE.0.00011)GO TO 12
      SUMN=SUMN+0.25*P(M)*(R(L)-R(M))*(R(M)+R(L))**2
      IF(M.GE.K)GO TO 12
      M=M+1
      GO TO 10
   12 V(J)=SUMN/SUMD
      SB(J)=V(J)/R(NY)**2
      SH(J)=V(J)/Z**2
      RETURN
      END
```

APPENDIX C

GRAND RIVER TESTS - OUTPUT OF MIXANDAT PROGRAM

C.1: JUNE 1975 TEST OUTPUT

The output of the MIXANDAT program of the June 12, 1975 test data is presented on the next five pages. A brief description of the output is given below.

On the following page, the title of the study and the results of data analysis at Transect A are printed. At this transect, there are three water quality parameters. For each parameter, the background concentration, flow rates and mass flux values are printed out, together with the mean depth, mean velocity and shape-velocity factor. The lateral distances, depths, velocities, concentrations, cumulative area, flow rate and mass flux values, and nondimensional values of flowrate and concentration are tabulated for each parameter. Finally, the variance values determined from the distributions of the three parameters are summarized. Similarly, the outputs for the other four transects are presented on the following pages. Note that there are only two water quality parameters at Transect B.

Units: All quantities are expressed in the metric units. The units of concentrations and mass flux are as follows (also see subsections 3.2.4 and 4.1.2):

Tracer	Concentration Units	Mass Flux Units
Rhodamine dye	ug/L	mg/s
Chloride ion	mg/L	gm/s
Conductivity	umho/cm	(umho-m^3)/(cm-sec)

WATERLOO STP MIXING STUDY JUNE 12,1975

TRANSECT A 121.9 METERS FROM OUTFALL
PARAMETER 1: RHODAMINE WT DYE
QRIVER= 12.544 BACKGROUND CONC.= 0.0
QEFL = 0.393 EFFLUENT CONCN.= 181.000
UPSTREAM FLUX= 0.0 EFFLUENT FLUX= 71.14 TOTAL FLUX = 71.14
VELOCITIES SIMULATED FROM RESISTANCE EQN.

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	51.20	0.0	0.0	0.0	0.0	0.0	9.028	3.635
1.52	0.74	0.39	50.20	0.56	0.11	5.61	0.028	0.009	8.852	3.564
6.10	0.99	0.48	39.60	4.51	1.83	83.02	0.111	0.146	6.983	2.812
13.72	0.71	0.38	17.00	10.99	4.63	162.15	0.250	0.369	2.998	1.207
18.29	0.61	0.35	3.60	14.01	5.73	173.49	0.333	0.457	0.635	0.256
22.86	0.70	0.38	1.10	17.00	6.82	176.03	0.417	0.543	0.194	0.078
27.43	0.66	0.37	0.0	20.11	7.97	176.67	0.500	0.636	0.0	0.0
32.00	0.61	0.35	0.0	23.01	9.01	176.67	0.583	0.718	0.0	0.0
36.58	0.62	0.35	0.0	25.83	9.99	176.67	0.667	0.796	0.0	0.0
41.15	0.62	0.35	0.0	28.67	10.99	176.67	0.750	0.876	0.0	0.0
45.72	0.52	0.31	0.0	31.28	11.85	176.67	0.833	0.945	0.0	0.0
50.29	0.38	0.25	0.0	33.35	12.43	176.67	0.917	0.991	0.0	0.0
54.86	0.0	0.0	0.0	34.22	12.54	176.67	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 5.671
AVG. CONC. AT THE TRANSECT, CATRN = 14.084 TOTAL FLUX AT TRANSECT= 176.67
MEAN DEPTH= 0.624 MEAN VELOCITY= 0.367 SHAPE-VELOCITY FACTOR= 1.409

TRANSECT A 121.9 METERS FROM OUTFALL
PARAMETER 2: CHLORIDE
QRIVER= 12.544 BACKGROUND CONC.= 12.000
QEFL = 0.393 EFFLUENT CONCN.= 270.000
UPSTREAM FLUX= 145.82 EFFLUENT FLUX= 106.12 TOTAL FLUX = 251.94
VELOCITIES SIMULATED FROM RESISTANCE EQN.

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	63.00	0.0	0.0	0.0	0.0	0.0	7.447	7.493
1.52	0.74	0.39	58.00	0.56	0.11	6.69	0.028	0.009	6.856	6.898
6.10	0.99	0.48	19.00	4.51	1.83	73.07	0.111	0.146	2.246	2.260
13.72	0.71	0.38	3.00	10.99	4.63	103.83	0.250	0.369	0.355	0.357
18.29	0.61	0.35	0.0	14.01	5.73	105.48	0.333	0.457	0.0	0.0
22.86	0.70	0.38	0.0	17.00	6.82	105.48	0.417	0.543	0.0	0.0
27.43	0.66	0.37	0.0	20.11	7.97	105.48	0.500	0.636	0.0	0.0
32.00	0.61	0.35	0.0	23.01	9.01	105.48	0.583	0.718	0.0	0.0
36.58	0.62	0.35	0.0	25.83	9.99	105.48	0.667	0.796	0.0	0.0
41.15	0.62	0.35	0.0	28.67	10.99	105.48	0.750	0.876	0.0	0.0
45.72	0.52	0.31	0.0	31.28	11.85	105.48	0.833	0.945	0.0	0.0
50.29	0.38	0.25	0.0	33.35	12.43	105.48	0.917	0.991	0.0	0.0
54.86	0.0	0.0	0.0	34.22	12.54	105.48	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 8.460
AVG. CONC. AT THE TRANSECT, CATRN = 8.408 TOTAL FLUX AT TRANSECT= 251.29

TRANSECT A 121.9 METERS FROM OUTFALL
PARAMETER 3: CONDUCTIVITY
QRIVER= 12.544 BACKGROUND CONC.= 395.000
QEFL = 0.393 EFFLUENT CONCN.= 1571.000
UPSTREAM FLUX= 4799.77 EFFLUENT FLUX= 617.46 TOTAL FLUX = 5417.23
VELOCITIES SIMULATED FROM RESISTANCE EQN.

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	325.00	0.0	0.0	0.0	0.0	0.0	6.603	6.652
1.52	0.74	0.39	300.00	0.56	0.11	34.55	0.028	0.009	6.095	6.141
6.10	0.99	0.48	115.00	4.51	1.83	392.31	0.111	0.146	2.336	2.354
13.72	0.71	0.38	25.00	10.99	4.63	588.03	0.250	0.369	0.508	0.512
18.29	0.61	0.35	5.00	14.01	5.73	604.55	0.333	0.457	0.102	0.102
22.86	0.70	0.38	5.00	17.00	6.82	609.97	0.417	0.543	0.102	0.102
27.43	0.66	0.37	0.0	20.11	7.97	612.86	0.500	0.636	0.0	0.0
32.00	0.61	0.35	0.0	23.01	9.01	612.86	0.583	0.718	0.0	0.0
36.58	0.62	0.35	0.0	25.83	9.99	612.86	0.667	0.796	0.0	0.0
41.15	0.62	0.35	0.0	28.67	10.99	612.86	0.750	0.876	0.0	0.0
45.72	0.52	0.31	0.0	31.28	11.85	612.86	0.833	0.945	0.0	0.0
50.29	0.38	0.25	0.0	33.35	12.43	612.86	0.917	0.991	0.0	0.0
54.86	0.0	0.0	0.0	34.22	12.54	612.86	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 49.222
AVG. CONC. AT THE TRANSECT, CATRN = 48.856 TOTAL FLUX AT TRANSECT= 5412.63

TRANSECT A PARAMETER	VCMAX	:VARIANCE FROM DIFFERENT METHODS: VCN	VUF	VCQ	VPQ
1	18.94	75.88	75.15	7.58	1.89
2	5.16	35.08	41.79	4.06	0.45
3	6.40	48.93	53.47	5.22	0.57

NONDIMENSIONAL VARIANCE X/B= 2.22 X/H= 195.5

PARAMETER	VCN/BB	VCN/HH	VUF/BB	VUF/HH	VCQ/QQ
1	0.0252	195.09	0.0250	193.21	0.0482
2	0.0117	90.19	0.0139	107.46	0.0258
3	0.0163	125.80	0.0178	137.49	0.0332

TRANSECT B   426.7 METERS FROM OUTFALL
PARAMETER 1: CHLORIDE
QRIVER=   12.581   BACKGROUND CONC.= 12.000
QEFL =   0.393   EFFLUENT CONCN.= 270.000
UPSTREAM FLUX=   146.25   EFFLUENT FLUX=   106.12   TOTAL FLUX =   252.37
MEASURED VELOCITIES CORRECTED TO GET Q=SUMQ(NYZ)

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	42.00	0.0	0.0	0.0	0.0	0.0	4.979	5.380
6.10	0.45	0.18	40.00	1.36	0.12	5.00	0.070	0.010	4.742	5.124
10.67	0.63	0.23	35.00	3.82	0.62	23.75	0.123	0.049	4.149	4.483
15.24	0.69	0.26	30.00	6.84	1.36	47.66	0.175	0.108	3.557	3.843
19.81	0.63	0.23	21.00	9.86	2.10	66.53	0.228	0.167	2.490	2.690
24.38	0.71	0.27	14.00	12.93	2.86	79.81	0.281	0.227	1.660	1.793
28.96	0.76	0.27	6.00	16.30	3.75	88.74	0.333	0.298	0.711	0.769
33.53	0.76	0.28	4.00	19.78	4.70	93.49	0.386	0.374	0.474	0.512
44.20	0.79	0.29	0.0	28.04	7.07	98.22	0.509	0.562	0.0	0.0
54.86	0.79	0.26	0.0	36.43	9.37	98.22	0.632	0.745	0.0	0.0
65.53	0.63	0.22	0.0	44.00	11.16	98.22	0.754	0.887	0.0	0.0
76.20	0.60	0.15	0.0	50.57	12.35	98.22	0.877	0.981	0.0	0.0
86.87	0.0	0.0	0.0	53.76	12.58	98.22	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG=   8.435
AVG. CONC. AT THE TRANSECT, CATRN =   7.807   TOTAL FLUX AT TRANSECT=   244.47
MEAN DEPTH= 0.619   MEAN VELOCITY= 0.234   SHAPE-VELOCITY FACTOR= 1.458

TRANSECT B   426.7 METERS FROM OUTFALL
PARAMETER 2: CONDUCTIVITY
QRIVER=   12.581   BACKGROUND CONC.= 390.000
QEFL =   0.393   EFFLUENT CONCN.= 1571.000
UPSTREAM FLUX= 4753.26   EFFLUENT FLUX=   617.46   TOTAL FLUX =   5370.72
MEASURED VELOCITIES CORRECTED TO GET Q=SUMQ(NYZ)

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	240.00	0.0	0.0	0.0	0.0	0.0	4.890	5.549
6.10	0.45	0.18	220.00	1.36	0.12	28.05	0.070	0.010	4.483	5.087
10.67	0.63	0.23	190.00	3.82	0.62	130.54	0.123	0.049	3.871	4.393
15.24	0.69	0.26	160.00	6.84	1.36	259.28	0.175	0.108	3.260	3.700
19.81	0.63	0.23	110.00	9.86	2.10	359.22	0.228	0.167	2.241	2.543
24.38	0.71	0.27	70.00	12.93	2.86	427.50	0.281	0.227	1.426	1.619
28.96	0.76	0.27	35.00	16.30	3.75	474.35	0.333	0.298	0.713	0.809
33.53	0.76	0.28	25.00	19.78	4.70	502.86	0.386	0.374	0.509	0.578
44.20	0.79	0.29	5.00	28.04	7.07	538.36	0.509	0.562	0.102	0.116
54.86	0.79	0.26	0.0	36.43	9.37	544.11	0.632	0.745	0.0	0.0
65.53	0.63	0.22	0.0	44.00	11.16	544.11	0.754	0.887	0.0	0.0
76.20	0.60	0.15	0.0	50.57	12.35	544.11	0.877	0.981	0.0	0.0
86.87	0.0	0.0	0.0	53.76	12.58	544.11	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 49.079
AVG. CONC. AT THE TRANSECT, CATRN = 43.249   TOTAL FLUX AT TRANSECT=   5297.36

TRANSECT B   :VARIANCE FROM DIFFERENT METHODS:

PARAMETER	VCMAX	VCN	VUF	VCQ	VPQ
1	66.18	234.96	364.39	5.19	0.87
2	61.87	256.84	407.96	6.42	0.82

NONDIMENSIONAL VARIANCE   X/B=   4.91   X/H=   689.5

PARAMETER	VCN/BB	VCN/HH	VUF/BB	VUF/HH	VCQ/QQ
1	0.0311	613.51	0.0483	951.47	0.0328
2	0.0340	670.64	0.0541	1065.25	0.0406

TRANSECT C 1082.0 METERS FROM OUTFALL
PARAMETER 1: RHODAMINE WT DYE
QRIVER= 12.601 BACKGROUND CONC.= 0.0
QEFL = 0.393 EFFLUENT CONCN.= 181.000
UPSTREAM FLUX= 0.0 EFFLUENT FLUX= 71.14 TOTAL FLUX = 71.14
VELOCITIES SIMULATED FROM RESISTANCE EQN.

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	2.63	0.0	0.0	0.0	0.0	0.0	0.466	0.698
3.05	0.23	0.29	6.94	0.35	0.05	0.24	0.069	0.004	1.229	1.843
7.62	0.41	0.43	23.10	1.80	0.58	8.12	0.172	0.046	4.092	6.134
12.19	0.53	0.52	13.80	3.94	1.59	26.89	0.276	0.126	2.444	3.665
16.76	0.59	0.55	5.28	6.50	2.96	39.92	0.379	0.235	0.935	1.402
21.34	0.71	0.63	1.40	9.46	4.70	45.74	0.483	0.373	0.248	0.372
24.38	0.75	0.65	0.47	11.69	6.12	47.07	0.552	0.486	0.083	0.125
27.43	0.84	0.70	0.0	14.11	7.76	47.45	0.621	0.616	0.0	0.0
30.48	0.75	0.65	0.0	16.53	9.39	47.45	0.690	0.745	0.0	0.0
35.05	0.61	0.57	0.0	19.64	11.28	47.45	0.793	0.895	0.0	0.0
39.62	0.39	0.42	0.0	21.93	12.41	47.45	0.897	0.985	0.0	0.0
44.20	0.0	0.0	0.0	22.83	12.60	47.45	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 5.646
AVG. CONC. AT THE TRANSECT, CATRN = 3.766 TOTAL FLUX AT TRANSECT= 47.45
MEAN DEPTH= 0.516 MEAN VELOCITY= 0.552 SHAPE-VELOCITY FACTOR= 1.795

TRANSECT C 1082.0 METERS FROM OUTFALL
PARAMETER 2: CHLORIDE
QRIVER= 12.601 BACKGROUND CONC.= 13.000
QEFL = 0.393 EFFLUENT CONCN.= 270.000
UPSTREAM FLUX= 158.70 EFFLUENT FLUX= 106.12 TOTAL FLUX = 264.82
VELOCITIES SIMULATED FROM RESISTANCE EQN.

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	75.00	0.0	0.0	0.0	0.0	0.0	8.906	7.682
3.05	0.23	0.29	69.00	0.35	0.05	3.68	0.069	0.004	8.193	7.068
7.62	0.41	0.43	36.00	1.80	0.58	31.21	0.172	0.046	4.275	3.688
12.19	0.53	0.52	25.00	3.94	1.59	62.24	0.276	0.126	2.969	2.561
16.76	0.59	0.55	17.00	6.50	2.96	90.91	0.379	0.235	2.019	1.741
21.34	0.71	0.63	7.00	9.46	4.70	111.83	0.483	0.373	0.831	0.717
24.38	0.75	0.65	3.00	11.69	6.12	118.93	0.552	0.486	0.356	0.307
27.43	0.84	0.70	1.00	14.11	7.76	122.20	0.621	0.616	0.119	0.102
30.48	0.75	0.65	0.0	16.53	9.39	123.02	0.690	0.745	0.0	0.0
35.05	0.61	0.57	0.0	19.64	11.28	123.02	0.793	0.895	0.0	0.0
39.62	0.39	0.42	0.0	21.93	12.41	123.02	0.897	0.985	0.0	0.0
44.20	0.0	0.0	0.0	22.83	12.60	123.02	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 8.422
AVG. CONC. AT THE TRANSECT, CATRN = 9.763 TOTAL FLUX AT TRANSECT= 281.72

TRANSECT C 1082.0 METERS FROM OUTFALL
PARAMETER 3: CONDUCTIVITY
QRIVER= 12.601 BACKGROUND CONC.= 395.000
QEFL = 0.393 EFFLUENT CONCN.= 1571.000
UPSTREAM FLUX= 4822.14 EFFLUENT FLUX= 617.46 TOTAL FLUX = 5439.60
VELOCITIES SIMULATED FROM RESISTANCE EQN.

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	350.00	0.0	0.0	0.0	0.0	0.0	7.143	6.748
3.05	0.23	0.29	325.00	0.35	0.05	17.24	0.069	0.004	6.633	6.266
7.62	0.41	0.43	205.00	1.80	0.58	156.23	0.172	0.046	4.184	3.952
12.19	0.53	0.52	140.00	3.94	1.59	331.69	0.276	0.126	2.857	2.699
16.76	0.59	0.55	90.00	6.50	2.96	488.72	0.379	0.235	1.837	1.735
21.34	0.71	0.63	35.00	9.46	4.70	597.68	0.483	0.373	0.714	0.675
24.38	0.75	0.65	15.00	11.69	6.12	633.19	0.552	0.486	0.306	0.289
27.43	0.84	0.70	5.00	14.11	7.76	649.53	0.621	0.616	0.102	0.096
30.48	0.75	0.65	0.0	16.53	9.39	653.62	0.690	0.745	0.0	0.0
35.05	0.61	0.57	0.0	19.64	11.28	653.62	0.793	0.895	0.0	0.0
39.62	0.39	0.42	0.0	21.93	12.41	653.62	0.897	0.985	0.0	0.0
44.20	0.0	0.0	0.0	22.83	12.60	653.62	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 49.001
AVG. CONC. AT THE TRANSECT, CATRN = 51.870 TOTAL FLUX AT TRANSECT= 5475.75

TRANSECT C :VARIANCE FROM DIFFERENT METHODS:

PARAMETER	VCMAX	VCN	VUF	VCQ	VPQ
1	15.79	117.01	165.68	4.91	0.67
2	16.89	96.28	195.33	7.48	0.43
3	20.32	99.84	193.72	7.28	0.56

NONDIMENSIONAL VARIANCE X/B= 24.48 X/H= 2095.0

PARAMETER	VCN/BB	VCN/HH	VUF/BB	VUF/HH	VCQ/QQ
1	0.0599	438.65	0.0848	621.11	0.0309
2	0.0493	360.93	0.1000	732.25	0.0471
3	0.0511	374.27	0.0992	726.23	0.0458

TRANSECT D 2240.3 METERS FROM OUTFALL
PARAMETER 1: RHODAMINE WT DYE
QRIVER= 12.629 BACKGROUND CONC.= 0.0
QEFL = 0.393 EFFLUENT CONCN.= 181.000
UPSTREAM FLUX= 0.0 EFFLUENT FLUX= 71.14 TOTAL FLUX = 71.14
MEASURED VELOCITIES CORRECTED TO GET Q=SUMQ(NYZ)

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	0.26	0.0	0.0	0.0	0.0	0.0	0.046	0.106
1.52	0.14	0.05	0.34	0.11	0.00	0.00	0.026	0.000	0.060	0.139
4.57	0.22	0.17	0.56	0.65	0.06	0.03	0.077	0.005	0.099	0.229
10.67	0.42	0.11	8.47	2.59	0.34	1.27	0.179	0.027	1.504	3.464
16.76	0.48	0.20	13.70	5.34	0.77	6.00	0.282	0.061	2.432	5.603
20.73	0.50	0.32	12.47	7.28	1.27	12.65	0.349	0.101	2.214	5.100
22.86	0.51	0.38	8.89	8.35	1.65	16.70	0.385	0.131	1.578	3.636
28.96	0.56	0.45	2.80	11.60	3.00	24.60	0.487	0.238	0.497	1.145
35.05	0.85	0.62	0.77	15.90	5.29	28.67	0.590	0.419	0.137	0.315
41.15	0.75	0.61	0.39	20.77	8.27	30.40	0.692	0.655	0.069	0.160
47.24	0.61	0.57	0.0	24.92	10.72	30.88	0.795	0.849	0.0	0.0
53.34	0.42	0.46	0.0	28.06	12.34	30.88	0.897	0.977	0.0	0.0
59.44	0.0	0.0	0.0	29.34	12.63	30.88	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 5.633
AVG. CONC. AT THE TRANSECT, CATRN = 2.445 TOTAL FLUX AT TRANSECT= 30.88
MEAN DEPTH= 0.494 MEAN VELOCITY= 0.430 SHAPE-VELOCITY FACTOR= 2.194

TRANSECT D 2240.3 METERS FROM OUTFALL
PARAMETER 2: CHLORIDE
QRIVER= 12.629 BACKGROUND CONC.= 13.000
QEFL = 0.393 EFFLUENT CONCN.= 270.000
UPSTREAM FLUX= 159.07 EFFLUENT FLUX= 106.12 TOTAL FLUX = 265.19
MEASURED VELOCITIES CORRECTED TO GET Q=SUMQ(NYZ)

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	49.00	0.0	0.0	0.0	0.0	0.0	5.831	6.495
1.52	0.14	0.05	45.00	0.11	0.00	0.13	0.026	0.000	5.355	5.965
4.57	0.22	0.17	42.00	0.65	0.06	2.79	0.077	0.005	4.998	5.567
10.67	0.42	0.11	33.00	2.59	0.34	13.11	0.179	0.027	3.927	4.374
16.76	0.48	0.20	25.00	5.34	0.77	25.49	0.282	0.061	2.975	3.314
20.73	0.50	0.32	21.00	7.28	1.27	37.17	0.349	0.101	2.499	2.784
22.86	0.51	0.38	19.00	8.35	1.65	44.75	0.385	0.131	2.261	2.518
28.96	0.56	0.45	12.00	11.60	3.00	65.71	0.487	0.238	1.428	1.591
35.05	0.85	0.62	5.00	15.90	5.29	85.10	0.590	0.419	0.595	0.663
41.15	0.75	0.61	1.00	20.77	8.27	94.05	0.692	0.655	0.119	0.133
47.24	0.61	0.57	0.0	24.92	10.72	95.28	0.795	0.849	0.0	0.0
53.34	0.42	0.46	0.0	28.06	12.34	95.28	0.897	0.977	0.0	0.0
59.44	0.0	0.0	0.0	29.34	12.63	95.28	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 8.403
AVG. CONC. AT THE TRANSECT, CATRN = 7.544 TOTAL FLUX AT TRANSECT= 254.35

TRANSECT D 2240.3 METERS FROM OUTFALL
PARAMETER 3: CONDUCTIVITY
QRIVER= 12.629 BACKGROUND CONC.= 390.000
QEFL = 0.393 EFFLUENT CONCN.= 1571.000
UPSTREAM FLUX= 4772.14 EFFLUENT FLUX= 617.46 TOTAL FLUX = 5389.61
MEASURED VELOCITIES CORRECTED TO GET Q=SUMQ(NYZ)

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	225.00	0.0	0.0	0.0	0.0	0.0	4.602	5.441
1.52	0.14	0.05	220.00	0.11	0.00	0.62	0.026	0.000	4.500	5.320
4.57	0.22	0.17	210.00	0.65	0.06	13.78	0.077	0.005	4.295	5.078
10.67	0.42	0.11	180.00	2.59	0.34	67.41	0.179	0.027	3.682	4.353
16.76	0.48	0.20	145.00	5.34	0.77	136.82	0.282	0.061	2.966	3.506
20.73	0.50	0.32	115.00	7.28	1.27	202.82	0.349	0.101	2.352	2.781
22.86	0.51	0.38	100.00	8.35	1.65	243.58	0.385	0.131	2.045	2.418
28.96	0.56	0.45	65.00	11.60	3.00	355.10	0.487	0.238	1.329	1.572
35.05	0.85	0.62	25.00	15.90	5.29	457.76	0.590	0.419	0.511	0.605
41.15	0.75	0.61	10.00	20.77	8.27	510.01	0.692	0.655	0.205	0.242
47.24	0.61	0.57	0.0	24.92	10.72	522.25	0.795	0.849	0.0	0.0
53.34	0.42	0.46	0.0	28.06	12.34	522.25	0.897	0.977	0.0	0.0
59.44	0.0	0.0	0.0	29.34	12.63	522.25	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 48.891
AVG. CONC. AT THE TRANSECT, CATRN = 41.352 TOTAL FLUX AT TRANSECT= 5294.39

TRANSECT D :VARIANCE FROM DIFFERENT METHODS:

PARAMETER	VCMAX	VCN	VUF	VCQ	VPQ
1	42.09	385.46	587.48	8.18	0.81
2	55.01	273.41	629.49	10.51	0.60
3	73.42	286.50	648.36	11.59	0.86

NONDIMENSIONAL VARIANCE X/B= 37.69 X/H= 4538.3

PARAMETER	VCN/BB	VCN/HH	VUF/BB	VUF/HH	VCQ/QQ
1	0.1091	1581.83	0.1663	2410.83	0.0513
2	0.0774	1122.00	0.1782	2583.24	0.0659
3	0.0811	1175.71	0.1835	2660.67	0.0727

TRANSECT E 3398.5 METERS FROM OUTFALL
PARAMETER 1: RHODAMINE WT DYE
QRIVER= 12.658 BACKGROUND CONC.= 0.0
QEFL = 0.393 EFFLUENT CONCN.= 181.000
UPSTREAM FLUX= 0.0 EFFLUENT FLUX= 71.14 TOTAL FLUX = 71.14
VELOCITIES SIMULATED FROM RESISTANCE EQN.

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	1.56	0.0	0.0	0.0	0.0	0.0	0.278	0.312
4.57	0.27	0.17	6.29	0.61	0.05	0.21	0.091	0.004	1.119	1.260
9.14	0.69	0.32	10.80	2.79	0.59	4.84	0.182	0.047	1.922	2.163
13.72	1.01	0.42	10.90	6.68	2.04	20.55	0.273	0.161	1.939	2.183
18.29	0.99	0.41	8.44	11.27	3.96	39.08	0.364	0.313	1.502	1.690
22.86	1.22	0.48	4.60	16.32	6.21	53.75	0.455	0.491	0.818	0.921
27.43	0.88	0.38	1.80	21.11	8.27	60.34	0.545	0.653	0.320	0.361
32.00	0.86	0.38	0.73	25.09	9.78	62.25	0.636	0.773	0.130	0.146
36.58	0.76	0.35	0.40	28.80	11.13	63.01	0.727	0.879	0.071	0.080
41.15	0.56	0.28	0.0	31.82	12.08	63.20	0.818	0.954	0.0	0.0
45.72	0.34	0.20	0.0	33.88	12.58	63.20	0.909	0.994	0.0	0.0
50.29	0.0	0.0	0.0	34.67	12.66	63.20	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 5.620
AVG. CONC. AT THE TRANSECT, CATRN = 4.993 TOTAL FLUX AT TRANSECT= 63.20
MEAN DEPTH= 0.689 MEAN VELOCITY= 0.365 SHAPE-VELOCITY FACTOR= 1.973

TRANSECT E 3398.5 METERS FROM OUTFALL
PARAMETER 2: CHLORIDE
QRIVER= 12.658 BACKGROUND CONC.= 13.000
QEFL = 0.393 EFFLUENT CONCN.= 270.000
UPSTREAM FLUX= 159.44 EFFLUENT FLUX= 106.12 TOTAL FLUX = 265.56
VELOCITIES SIMULATED FROM RESISTANCE EQN.

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	28.00	0.0	0.0	0.0	0.0	0.0	3.340	2.680
4.57	0.27	0.17	26.00	0.61	0.05	1.43	0.091	0.004	3.101	2.489
9.14	0.69	0.32	27.00	2.79	0.59	15.78	0.182	0.047	3.220	2.585
13.72	1.01	0.42	23.00	6.68	2.04	52.00	0.273	0.161	2.743	2.202
18.29	0.99	0.41	15.00	11.27	3.96	88.40	0.364	0.313	1.789	1.436
22.86	1.22	0.48	8.00	16.32	6.21	114.28	0.455	0.491	0.954	0.766
27.43	0.88	0.38	4.00	21.11	8.27	126.62	0.545	0.653	0.477	0.383
32.00	0.86	0.38	1.00	25.09	9.78	130.40	0.636	0.773	0.119	0.096
36.58	0.76	0.35	1.00	28.80	11.13	131.75	0.727	0.879	0.119	0.096
41.15	0.56	0.28	0.0	31.82	12.08	132.22	0.818	0.954	0.0	0.0
45.72	0.34	0.20	0.0	33.88	12.58	132.22	0.909	0.994	0.0	0.0
50.29	0.0	0.0	0.0	34.67	12.66	132.22	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 8.384
AVG. CONC. AT THE TRANSECT, CATRN = 10.446 TOTAL FLUX AT TRANSECT= 291.66

TRANSECT E 3398.5 METERS FROM OUTFALL
PARAMETER 3: CONDUCTIVITY
QRIVER= 12.658 BACKGROUND CONC.= 390.000
QEFL = 0.393 EFFLUENT CONCN.= 1571.000
UPSTREAM FLUX= 4783.19 EFFLUENT FLUX= 617.46 TOTAL FLUX = 5400.65
VELOCITIES SIMULATED FROM RESISTANCE EQN.

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	160.00	0.0	0.0	0.0	0.0	0.0	3.280	2.868
4.57	0.27	0.17	150.00	0.61	0.05	8.21	0.091	0.004	3.075	2.689
9.14	0.69	0.32	150.00	2.79	0.59	89.45	0.182	0.047	3.075	2.689
13.72	1.01	0.42	120.00	6.68	2.04	285.02	0.273	0.161	2.460	2.151
18.29	0.99	0.41	85.00	11.27	3.96	481.40	0.364	0.313	1.742	1.524
22.86	1.22	0.48	40.00	16.32	6.21	622.02	0.455	0.491	0.820	0.717
27.43	0.88	0.38	20.00	21.11	8.27	683.76	0.545	0.653	0.410	0.359
32.00	0.86	0.38	5.00	25.09	9.78	702.65	0.636	0.773	0.102	0.090
36.58	0.76	0.35	0.0	28.80	11.13	706.02	0.727	0.879	0.0	0.0
41.15	0.56	0.28	0.0	31.82	12.08	706.02	0.818	0.954	0.0	0.0
45.72	0.34	0.20	0.0	33.88	12.58	706.02	0.909	0.994	0.0	0.0
50.29	0.0	0.0	0.0	34.67	12.66	706.02	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 48.782
AVG. CONC. AT THE TRANSECT, CATRN = 55.778 TOTAL FLUX AT TRANSECT= 5489.21

TRANSECT E : VARIANCE FROM DIFFERENT METHODS :

PARAMETER	VCMAX	VCN	VUF	VCQ	VPQ
1	56.05	249.95	320.58	18.72	5.35
2	60.09	187.56	295.97	16.84	3.55
3	54.90	171.00	281.73	15.43	3.10

NONDIMENSIONAL VARIANCE X/B= 67.58 X/H= 4929.7

PARAMETER	VCN/BB	VCN/HH	VUF/BB	VUF/HH	VLQ/QQ
1	0.0988	525.91	0.1267	674.52	0.1169
2	0.0742	394.63	0.1170	622.73	0.1051
3	0.0676	359.80	0.1114	592.78	0.0963

C.2: AUGUST 1975 TEST OUTPUT

The output of the August 12, 1975 field test data is presented on the following five pages. These output tables are similar to those of the June test.

WATERLOO STP EFFLUENT MIXING STUDY AUGUST 12, 1975

TRANSECT A 182.9 METERS FROM OUTFALL
PARAMETER 1: RHODAMINE WT DYE
QRIVER= 10.102 BACKGROUND CONC.= 0.0
QEFL = 0.341 EFFLUENT CONCN.= 98.000
UPSTREAM FLUX= 0.0 EFFLUENT FLUX= 33.38 TOTAL FLUX = 33.38
VELOCITIES SIMULATED FROM RESISTANCE EQN.

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	18.30	0.0	0.0	0.0	0.0	0.0	5.538	6.031
1.52	0.33	0.20	17.35	0.25	0.03	0.45	0.024	0.002	5.250	5.718
4.57	0.65	0.32	14.88	1.73	0.41	6.72	0.071	0.041	4.503	4.904
10.67	0.76	0.36	8.17	6.04	1.87	23.53	0.167	0.185	2.472	2.693
16.76	0.61	0.31	1.12	10.22	3.26	29.98	0.262	0.323	0.339	0.369
22.86	0.64	0.32	0.0	14.03	4.45	30.65	0.357	0.441	0.0	0.0
28.96	0.59	0.30	0.0	17.76	5.61	30.65	0.452	0.555	0.0	0.0
35.05	0.60	0.30	0.0	21.37	6.69	30.65	0.548	0.663	0.0	0.0
41.15	0.55	0.29	0.0	24.85	7.72	30.65	0.643	0.764	0.0	0.0
45.72	0.59	0.30	0.0	27.45	8.48	30.65	0.714	0.840	0.0	0.0
50.29	0.52	0.28	0.0	29.99	9.22	30.65	0.786	0.913	0.0	0.0
54.86	0.47	0.26	0.0	32.26	9.82	30.65	0.857	0.973	0.0	0.0
64.01	0.0	0.0	0.0	34.40	10.10	30.65	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 3.305
AVG. CONC. AT THE TRANSECT, CATRN = 3.034 TOTAL FLUX AT TRANSECT= 30.65
MEAN DEPTH= 0.537 MEAN VELOCITY= 0.294 SHAPE-VELOCITY FACTOR= 1.354

TRANSECT A 182.9 METERS FROM OUTFALL
PARAMETER 2: CHLORIDE
QRIVER= 10.102 BACKGROUND CONC.= 12.000
QEFL = 0.341 EFFLUENT CONCN.= 358.000
UPSTREAM FLUX= 117.14 EFFLUENT FLUX= 121.95 TOTAL FLUX = 239.09
VELOCITIES SIMULATED FROM RESISTANCE EQN.

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	78.00	0.0	0.0	0.0	0.0	0.0	6.461	6.466
1.52	0.33	0.20	76.00	0.25	0.03	1.94	0.024	0.002	6.295	6.301
4.57	0.65	0.32	63.00	1.73	0.41	28.97	0.071	0.041	5.219	5.223
10.67	0.76	0.36	25.00	6.04	1.87	93.15	0.167	0.185	2.071	2.073
16.76	0.61	0.31	7.00	10.22	3.26	115.39	0.262	0.323	0.580	0.580
22.86	0.64	0.32	1.00	14.03	4.45	120.16	0.357	0.441	0.083	0.083
28.96	0.59	0.30	1.00	17.76	5.61	121.31	0.452	0.555	0.083	0.083
35.05	0.60	0.30	0.0	21.37	6.69	121.86	0.548	0.663	0.0	0.0
41.15	0.55	0.29	0.0	24.85	7.72	121.86	0.643	0.764	0.0	0.0
45.72	0.59	0.30	0.0	27.45	8.48	121.86	0.714	0.840	0.0	0.0
50.29	0.52	0.28	0.0	29.99	9.22	121.86	0.786	0.913	0.0	0.0
54.86	0.47	0.26	0.0	32.26	9.82	121.86	0.857	0.973	0.0	0.0
64.01	0.0	0.0	0.0	34.40	10.10	121.86	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 12.072
AVG. CONC. AT THE TRANSECT, CATRN = 12.062 TOTAL FLUX AT TRANSECT= 238.99

TRANSECT A 182.9 METERS FROM OUTFALL
PARAMETER 3: CONDUCTIVITY
QRIVER= 10.102 BACKGROUND CONC.= 360.000
QEFL = 0.341 EFFLUENT CONCN.= 2257.000
UPSTREAM FLUX= 3514.10 EFFLUENT FLUX= 768.85 TOTAL FLUX = 4282.95
VELOCITIES SIMULATED FROM RESISTANCE EQN.

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	430.00	0.0	0.0	0.0	0.0	0.0	5.650	5.661
1.52	0.33	0.20	420.00	0.25	0.03	10.69	0.024	0.002	5.518	5.529
4.57	0.65	0.32	380.00	1.73	0.41	166.26	0.071	0.041	4.993	5.002
10.67	0.76	0.36	160.00	6.04	1.87	560.12	0.167	0.185	2.102	2.106
16.76	0.61	0.31	45.00	10.22	3.26	702.61	0.262	0.323	0.591	0.592
22.86	0.64	0.32	10.00	14.03	4.45	735.37	0.357	0.441	0.131	0.132
28.96	0.59	0.30	10.00	17.76	5.61	746.90	0.452	0.555	0.131	0.132
35.05	0.60	0.30	10.00	21.37	6.69	757.77	0.548	0.663	0.131	0.132
41.15	0.55	0.29	5.00	24.85	7.72	765.46	0.643	0.764	0.066	0.066
45.72	0.59	0.30	0.0	27.45	8.48	767.37	0.714	0.840	0.0	0.0
50.29	0.52	0.28	0.0	29.99	9.22	767.37	0.786	0.913	0.0	0.0
54.86	0.47	0.26	0.0	32.26	9.82	767.37	0.857	0.973	0.0	0.0
64.01	0.0	0.0	0.0	34.40	10.10	767.37	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 76.108
AVG. CONC. AT THE TRANSECT, CATRN = 75.962 TOTAL FLUX AT TRANSECT= 4281.46

TRANSECT A — VARIANCE FROM DIFFERENT METHODS:

PARAMETER	VCMAX	VCN	VUF	VCQ	VPQ
1	15.10	62.93	81.93	2.44	0.45
2	13.88	72.40	93.26	2.89	0.39
3	17.31	107.46	127.22	4.14	0.51

NONDIMENSIONAL VARIANCE X/B= 2.86 X/H= 340.3

PARAMETER	VCN/BB	VCN/HH	VUF/BB	VUF/HH	VCQ/QQ
1	0.0154	217.85	0.0200	283.62	0.0239
2	0.0177	250.65	0.0228	322.85	0.0284
3	0.0262	372.02	0.0311	440.42	0.0406

TRANSECT B 533.4 METERS FROM OUTFALL

PARAMETER 1: RHODAMINE WT DYE

QRIVER= 10.107 BACKGROUND CONC.= 0.0

QEFL = 0.341 EFFLUENT CONCN.= 98.000

UPSTREAM FLUX= 0.0 EFFLUENT FLUX= 33.38 TOTAL FLUX = 33.38

MEASURED VELOCITIES CORRECTED TO GET Q=SUMQ(NYZ)

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	9.00	0.0	0.0	0.0	0.0	0.0	2.725	3.324
1.52	0.12	0.04	10.07	0.09	0.00	0.02	0.019	0.000	3.049	3.719
3.05	0.24	0.07	11.35	0.37	0.02	0.18	0.037	0.002	3.436	4.192
7.47	0.47	0.13	13.49	1.94	0.18	2.20	0.091	0.018	4.084	4.982
10.67	0.70	0.20	12.63	3.81	0.49	6.25	0.130	0.048	3.824	4.665
18.29	0.65	0.22	8.36	8.94	1.56	17.50	0.222	0.155	2.531	3.088
25.91	0.69	0.25	3.02	14.03	2.76	24.29	0.315	0.273	0.914	1.115
33.53	0.79	0.21	0.88	19.64	4.04	26.79	0.407	0.399	0.266	0.325
37.19	0.77	0.23	0.45	22.49	4.66	27.20	0.452	0.461	0.136	0.166
41.15	0.76	0.25	0.0	25.53	5.37	27.36	0.500	0.532	0.0	0.0
48.77	0.79	0.22	0.0	31.43	6.75	27.36	0.593	0.668	0.0	0.0
56.39	0.76	0.27	0.0	37.31	8.17	27.36	0.685	0.809	0.0	0.0
64.01	0.67	0.19	0.0	42.75	9.42	27.36	0.778	0.932	0.0	0.0
71.63	0.48	0.08	0.0	47.16	10.01	27.36	0.870	0.990	0.0	0.0
82.30	0.0	0.0	0.0	49.72	10.11	27.36	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 3.303

AVG. CONC. AT THE TRANSECT, CATRN = 2.708 TOTAL FLUX AT TRANSECT= 27.36

MEAN DEPTH= 0.604 MEAN VELOCITY= 0.203 SHAPE-VELOCITY FACTOR= 1.541

TRANSECT B 533.4 METERS FROM OUTFALL

PARAMETER 2: CHLORIDE

QRIVER= 10.107 BACKGROUND CONC.= 12.000

QEFL = 0.341 EFFLUENT CONCN.= 358.000

UPSTREAM FLUX= 117.19 EFFLUENT FLUX= 121.95 TOTAL FLUX = 239.14

MEASURED VELOCITIES CORRECTED TO GET Q=SUMQ(NYZ)

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	45.00	0.0	0.0	0.0	0.0	0.0	3.729	4.727
1.52	0.12	0.04	44.00	0.09	0.00	0.08	0.019	0.000	3.646	4.622
3.05	0.24	0.07	43.00	0.37	0.02	0.74	0.037	0.002	3.564	4.517
7.47	0.47	0.13	40.00	1.94	0.18	7.48	0.091	0.018	3.315	4.202
10.67	0.70	0.20	38.00	3.81	0.49	19.59	0.130	0.048	3.149	3.992
18.29	0.65	0.22	30.00	8.94	1.56	56.02	0.222	0.155	2.486	3.152
25.91	0.69	0.25	13.00	14.03	2.76	81.69	0.315	0.273	1.077	1.366
33.53	0.79	0.21	4.00	19.64	4.04	92.59	0.407	0.399	0.331	0.420
37.19	0.77	0.23	2.00	22.49	4.66	94.44	0.452	0.461	0.166	0.210
41.15	0.76	0.25	1.00	25.53	5.37	95.52	0.500	0.532	0.083	0.105
48.77	0.79	0.22	0.0	31.43	6.75	96.21	0.593	0.668	0.0	0.0
56.39	0.76	0.27	0.0	37.31	8.17	96.21	0.685	0.809	0.0	0.0
64.01	0.67	0.19	0.0	42.75	9.42	96.21	0.778	0.932	0.0	0.0
71.63	0.48	0.08	0.0	47.16	10.01	96.21	0.870	0.990	0.0	0.0
82.30	0.0	0.0	0.0	49.72	10.11	96.21	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 12.067

AVG. CONC. AT THE TRANSECT, CATRN = 9.519 TOTAL FLUX AT TRANSECT= 213.40

TRANSECT B 533.4 METERS FROM OUTFALL

PARAMETER 3: CONDUCTIVITY

QRIVER= 10.107 BACKGROUND CONC.= 355.000

QEFL = 0.341 EFFLUENT CONCN.= 2257.000

UPSTREAM FLUX= 3466.90 EFFLUENT FLUX= 768.85 TOTAL FLUX = 4235.75

MEASURED VELOCITIES CORRECTED TO GET Q=SUMQ(NYZ)

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	295.00	0.0	0.0	0.0	0.0	0.0	3.878	4.584
1.52	0.12	0.04	290.00	0.09	0.00	0.50	0.019	0.000	3.812	4.506
3.05	0.24	0.07	285.00	0.37	0.02	4.86	0.037	0.002	3.746	4.429
7.47	0.47	0.13	270.00	1.94	0.18	49.99	0.091	0.018	3.549	4.195
10.67	0.70	0.20	245.00	3.81	0.49	129.89	0.130	0.048	3.221	3.807
18.29	0.65	0.22	165.00	8.94	1.56	349.58	0.222	0.155	2.169	2.564
25.91	0.69	0.25	90.00	14.03	2.76	501.79	0.315	0.273	1.183	1.398
33.53	0.79	0.21	35.00	19.64	4.04	581.93	0.407	0.399	0.460	0.544
37.19	0.77	0.23	25.00	22.49	4.66	600.45	0.452	0.461	0.329	0.388
41.15	0.76	0.25	15.00	25.53	5.37	614.83	0.500	0.532	0.197	0.233
48.77	0.79	0.22	10.00	31.43	6.75	632.03	0.593	0.668	0.131	0.155
56.39	0.76	0.27	5.00	37.31	8.17	642.71	0.685	0.809	0.066	0.078
64.01	0.67	0.19	5.00	42.75	9.42	648.93	0.778	0.932	0.066	0.078
71.63	0.48	0.08	0.0	47.16	10.01	650.41	0.870	0.990	0.0	0.0
82.30	0.0	0.0	0.0	49.72	10.11	650.41	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 76.074

AVG. CONC. AT THE TRANSECT, CATRN = 64.355 TOTAL FLUX AT TRANSECT= 4117.30

TRANSECT B VARIANCE FROM DIFFERENT METHODS:

PARAMETER	VCMAX	VCN	VUF	VCQ	VPQ
1	63.31	224.92	330.15	3.09	0.65
2	71.04	250.08	377.75	3.87	0.73
3	73.57	340.79	519.41	6.82	0.77

NONDIMENSIONAL VARIANCE X/B= 6.48 X/H= 882.8

PARAMETER	VCN/BB	VCN/HH	VUF/BB	VUF/HH	VCQ/QQ
1	0.0332	616.08	0.0487	904.34	0.0303
2	0.0369	685.02	0.0558	1034.71	0.0379
3	0.0563	933.49	0.0767	1422.76	0.0667

TRANSECT C 1082.0 METERS FROM OUTFALL
PARAMETER 1: RHODAMINE WT DYE
QRIVER= 10.114 BACKGROUND CONC.= 0.0
QEFL = 0.341 EFFLUENT CONCN.= 98.000
UPSTREAM FLUX= 0.0 EFFLUENT FLUX= 33.38 TOTAL FLUX = 33.38
VELOCITIES SIMULATED FROM RESISTANCE EQN.

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	8.79	0.0	0.0	0.0	0.0	0.0	2.663	9.269
3.05	0.25	0.27	8.36	0.39	0.05	0.45	0.071	0.005	2.533	8.816
6.10	0.32	0.32	7.50	1.26	0.31	2.52	0.143	0.031	2.272	7.909
9.14	0.45	0.40	5.06	2.43	0.74	5.17	0.214	0.073	1.533	5.336
13.72	0.50	0.43	1.42	4.59	1.63	8.05	0.321	0.161	0.430	1.497
18.29	0.65	0.51	0.45	7.21	2.86	9.21	0.429	0.283	0.136	0.475
22.86	0.74	0.56	0.0	10.38	4.56	9.59	0.536	0.451	0.0	0.0
27.43	0.79	0.58	0.0	13.86	6.55	9.59	0.643	0.648	0.0	0.0
32.00	0.65	0.51	0.0	17.14	8.35	9.59	0.750	0.826	0.0	0.0
36.58	0.46	0.41	0.0	19.67	9.51	9.59	0.857	0.941	0.0	0.0
41.15	0.27	0.28	0.0	21.33	10.09	9.59	0.964	0.997	0.0	0.0
42.67	0.0	0.0	0.0	21.53	10.11	9.59	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 3.301
AVG. CONC. AT THE TRANSECT, CATRN = 0.948 TOTAL FLUX AT TRANSECT= 9.59
MEAN DEPTH= 0.505 MEAN VELOCITY= 0.470 SHAPE-VELOCITY FACTOR= 1.735

TRANSECT C 1082.0 METERS FROM OUTFALL
PARAMETER 2: CHLORIDE
QRIVER= 10.114 BACKGROUND CONC.= 12.000
QEFL = 0.341 EFFLUENT CONCN.= 358.000
UPSTREAM FLUX= 117.28 EFFLUENT FLUX= 121.95 TOTAL FLUX = 239.24
VELOCITIES SIMULATED FROM RESISTANCE EQN.

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	88.00	0.0	0.0	0.0	0.0	0.0	7.298	7.822
3.05	0.25	0.27	73.00	0.39	0.05	4.24	0.071	0.005	6.054	6.488
6.10	0.32	0.32	53.00	1.26	0.31	20.68	0.143	0.031	4.396	4.711
9.14	0.45	0.40	24.00	2.43	0.74	36.91	0.214	0.073	1.990	2.133
13.72	0.50	0.43	28.00	4.59	1.63	60.07	0.321	0.161	2.322	2.489
18.29	0.65	0.51	16.00	7.21	2.86	87.23	0.429	0.283	1.327	1.422
22.86	0.74	0.56	6.00	10.38	4.56	105.93	0.536	0.451	0.498	0.533
27.43	0.79	0.58	1.00	13.86	6.55	112.89	0.643	0.648	0.083	0.089
32.00	0.65	0.51	0.0	17.14	8.35	113.79	0.750	0.826	0.0	0.0
36.58	0.46	0.41	0.0	19.67	9.51	113.79	0.857	0.941	0.0	0.0
41.15	0.27	0.28	0.0	21.33	10.09	113.79	0.964	0.997	0.0	0.0
42.67	0.0	0.0	0.0	21.53	10.11	113.79	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 12.058
AVG. CONC. AT THE TRANSECT, CATRN = 11.251 TOTAL FLUX AT TRANSECT= 231.08

TRANSECT C 1082.0 METERS FROM OUTFALL
PARAMETER 3: CONDUCTIVITY
QRIVER= 10.114 BACKGROUND CONC.= 350.000
QEFL = 0.341 EFFLUENT CONCN.= 2257.000
UPSTREAM FLUX= 3420.74 EFFLUENT FLUX= 768.85 TOTAL FLUX = 4189.59
VELOCITIES SIMULATED FROM RESISTANCE EQN.

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	430.00	0.0	0.0	0.0	0.0	0.0	5.657	5.355
3.05	0.25	0.27	390.00	0.39	0.05	21.58	0.071	0.005	5.130	4.857
6.10	0.32	0.32	310.00	1.26	0.31	112.93	0.143	0.031	4.078	3.861
9.14	0.45	0.40	230.00	2.43	0.74	226.73	0.214	0.073	3.026	2.864
13.72	0.50	0.43	180.00	4.59	1.63	409.37	0.321	0.161	2.368	2.242
18.29	0.65	0.51	100.00	7.21	2.86	582.18	0.429	0.283	1.315	1.245
22.86	0.74	0.56	50.00	10.38	4.56	709.68	0.536	0.451	0.658	0.623
27.43	0.79	0.58	20.00	13.86	6.55	779.35	0.643	0.648	0.263	0.249
32.00	0.65	0.51	10.00	17.14	8.35	806.35	0.750	0.826	0.132	0.125
36.58	0.46	0.41	0.0	19.67	9.51	812.16	0.857	0.941	0.0	0.0
41.15	0.27	0.28	0.0	21.33	10.09	812.16	0.964	0.997	0.0	0.0
42.67	0.0	0.0	0.0	21.53	10.11	812.16	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 76.017
AVG. CONC. AT THE TRANSECT, CATRN = 80.299 TOTAL FLUX AT TRANSECT= 4232.91

TRANSECT C VARIANCE FROM DIFFERENT METHODS:

PARAMETER	VCMAX	VCN	VUF	VCQ	VPQ
1	16.52	57.42	107.66	1.66	0.19
2	14.60	102.17	214.35	6.12	0.27
3	24.28	130.50	252.88	8.65	0.57

NONDIMENSIONAL VARIANCE X/B= 25.36 X/H= 2144.1

PARAMETER	VCN/BB	VCN/HH	VUF/BB	VUF/HH	VCQ/QQ
1	0.0315	225.45	0.0591	422.72	0.0162
2	0.0561	401.18	0.1177	841.63	0.0598
3	0.0717	512.41	0.1389	992.93	0.0846

TRANSECT D 2240.3 METERS FROM OUTFALL
PARAMETER 1 RHODAMINE WT DYE
QRIVER= 10.125 BACKGROUND CONC.= 0.0
QEFL = 0.341 EFFLUENT CONCN.= 98.000
UPSTREAM FLUX= 0.0 EFFLUENT FLUX= 33.38 TOTAL FLUX = 33.38
MEASURED VELOCITIES CORRECTED TO GET Q=SUMQ(NYZ)

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	0.45	0.0	0.0	0.0	0.0	0.0	0.136	0.293
3.05	0.33	0.13	2.59	0.50	0.03	0.05	0.057	0.003	0.786	1.689
7.62	0.38	0.12	5.69	2.13	0.24	0.90	0.143	0.024	1.726	3.710
12.19	0.48	0.21	6.65	4.10	0.57	2.92	0.229	0.056	2.017	4.335
13.72	0.48	0.24	6.86	4.83	0.73	4.03	0.257	0.072	2.081	4.472
16.76	0.48	0.29	6.22	6.30	1.12	6.56	0.314	0.110	1.887	4.055
21.34	0.57	0.30	4.09	8.71	1.83	10.22	0.400	0.181	1.240	2.666
25.91	0.71	0.38	1.52	11.64	2.83	13.03	0.486	0.279	0.461	0.991
32.00	0.73	0.52	0.45	16.02	4.80	14.97	0.600	0.474	0.136	0.293
38.10	0.79	0.55	0.0	20.63	7.28	15.53	0.714	0.719	0.0	0.0
44.20	0.59	0.43	0.0	24.81	9.32	15.53	0.829	0.920	0.0	0.0
50.29	0.15	0.26	0.0	27.06	10.09	15.53	0.943	0.997	0.0	0.0
53.34	0.0	0.0	0.0	27.29	10.13	15.53	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 3.297
AVG. CONC. AT THE TRANSECT, CATRN = 1.534 TOTAL FLUX AT TRANSECT= 15.53
MEAN DEPTH= 0.512 MEAN VELOCITY= 0.371 SHAPE-VELOCITY FACTOR= 2.001

TRANSECT D 2240.3 METERS FROM OUTFALL
PARAMETER 2: CHLORIDE
QRIVER= 10.125 BACKGROUND CONC.= 12.000
QEFL = 0.341 EFFLUENT CONCN.= 358.000
UPSTREAM FLUX= 117.42 EFFLUENT FLUX= 121.95 TOTAL FLUX = 239.37
MEASURED VELOCITIES CORRECTED TO GET Q=SUMQ(NYZ)

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	46.00	0.0	0.0	0.0	0.0	0.0	3.819	4.621
3.05	0.33	0.13	43.00	0.50	0.03	1.49	0.057	0.003	3.570	4.320
7.62	0.38	0.12	38.00	2.13	0.24	9.79	0.143	0.024	3.155	3.817
12.19	0.48	0.21	24.00	4.10	0.57	19.93	0.229	0.056	1.993	2.411
13.72	0.48	0.24	26.00	4.83	0.73	24.06	0.257	0.072	2.159	2.612
16.76	0.48	0.29	30.00	6.30	1.12	34.87	0.314	0.110	2.491	3.014
21.34	0.57	0.30	22.00	8.71	1.83	53.36	0.400	0.181	1.827	2.210
25.91	0.71	0.38	14.00	11.64	2.83	71.35	0.486	0.279	1.162	1.406
32.00	0.73	0.52	6.00	16.02	4.80	91.11	0.600	0.474	0.498	0.603
38.10	0.79	0.55	1.00	20.63	7.28	99.77	0.714	0.719	0.083	0.100
44.20	0.59	0.43	0.0	24.81	9.32	100.79	0.829	0.920	0.0	0.0
50.29	0.15	0.26	0.0	27.06	10.09	100.79	0.943	0.997	0.0	0.0
53.34	0.0	0.0	0.0	27.29	10.13	100.79	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 12.044
AVG. CONC. AT THE TRANSECT, CATRN = 9.955 TOTAL FLUX AT TRANSECT= 218.21

TRANSECT D 2240.3 METERS FROM OUTFALL
PARAMETER 3: CONDUCTIVITY
QRIVER= 10.125 BACKGROUND CONC.= 350.000
QEFL = 0.341 EFFLUENT CONCN.= 2257.000
UPSTREAM FLUX= 3424.61 EFFLUENT FLUX= 768.85 TOTAL FLUX = 4193.46
MEASURED VELOCITIES CORRECTED TO GET Q=SUMQ(NYZ)

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	250.00	0.0	0.0	0.0	0.0	0.0	3.292	3.485
3.05	0.33	0.13	250.00	0.50	0.03	8.39	0.057	0.003	3.292	3.485
7.62	0.38	0.12	250.00	2.13	0.24	59.62	0.143	0.024	3.292	3.485
12.19	0.48	0.21	230.00	4.10	0.57	138.07	0.229	0.056	3.029	3.206
13.72	0.48	0.24	210.00	4.83	0.73	174.41	0.257	0.072	2.766	2.927
16.76	0.48	0.29	170.00	6.30	1.12	247.79	0.314	0.110	2.239	2.370
21.34	0.57	0.30	135.00	8.71	1.83	356.26	0.400	0.181	1.778	1.882
25.91	0.71	0.38	95.00	11.64	2.83	471.18	0.486	0.279	1.251	1.324
32.00	0.73	0.52	45.00	16.02	4.80	609.53	0.600	0.474	0.593	0.627
38.10	0.79	0.55	20.00	20.63	7.28	689.93	0.714	0.719	0.263	0.279
44.20	0.59	0.43	10.00	24.81	9.32	720.54	0.829	0.920	0.132	0.139
50.29	0.15	0.26	5.00	27.06	10.09	726.36	0.943	0.997	0.066	0.070
53.34	0.0	0.0	0.0	27.29	10.13	726.44	1.000	1.000	0.0	0.0

AVG. CONC. JUST BELOW OUTFALL, CAVG= 75.934
AVG. CONC. AT THE TRANSECT, CATRN = 71.745 TOTAL FLUX AT TRANSECT= 4151.04

TRANSECT D VARIANCE FROM DIFFERENT METHODS

PARAMETER	VCMAX	VCN	VUF	VCQ	VPQ
1	53.49	255.26	399.65	4.81	0.82
2	57.25	249.00	497.29	8.18	0.76
3	88.92	297.44	572.13	11.77	1.34

NONDIMENSIONAL VARIANCE X/B= 42.00 X/H= 4379.1

PARAMETER	VCN/BB	VCN/HH	VUF/BB	VUF/HH	VCQ/QQ
1	0.0897	975.32	0.1405	1527.02	0.0469
2	0.0875	951.42	0.1748	1900.10	0.0798
3	0.1045	1136.49	0.2011	2186.04	0.1148

```
TRANSECT E                 3429.0 METERS FROM OUTFALL
     PARAMETER 1: RHODAMINE WT DYE
QRIVER=    10.139     BACKGROUND CONC.=   0.0
QEFL   =    0.341     EFFLUENT CONCN.=   98.000
UPSTREAM FLUX=     0.0    EFFLUENT FLUX=     33.38   TOTAL FLUX =      33.38
     VELOCITIES SIMULATED FROM RESISTANCE EQN.
```

Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	0.98	0.0	0.0	0.0	0.0	0.0	0.298	0.499
1.83	0.12	0.23	1.13	0.11	0.01	0.01	0.050	0.001	0.343	0.575
3.05	0.24	0.37	1.24	0.33	0.08	0.09	0.083	0.008	0.377	0.631
6.10	0.39	0.51	1.83	1.30	0.50	0.74	0.167	0.050	0.556	0.931
9.14	0.48	0.58	2.85	2.63	1.23	2.45	0.250	0.121	0.866	1.450
10.36	0.57	0.65	3.02	3.27	1.62	3.60	0.283	0.160	0.917	1.537
12.19	0.65	0.71	3.23	4.38	2.38	5.97	0.333	0.235	0.981	1.644
15.24	0.79	0.81	2.70	6.57	4.05	10.91	0.417	0.399	0.820	1.374
18.29	0.79	0.81	1.96	8.97	5.99	15.44	0.500	0.591	0.595	0.997
21.34	0.80	0.82	1.11	11.39	7.96	18.46	0.583	0.786	0.337	0.565
24.38	0.08	0.18	0.92	12.74	8.64	19.15	0.667	0.852	0.279	0.468
28.96	0.08	0.17	0.81	13.10	8.70	19.20	0.792	0.858	0.246	0.412
30.48	0.84	0.85	0.80	13.80	9.06	19.49	0.833	0.893	0.243	0.407
36.58	0.0	0.0	0.0	16.36	10.14	19.92	1.000	1.000	0.0	0.0

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AVG. CONC. JUST BELOW OUTFALL, CAVG=    3.293
AVG. CONC. AT THE TRANSECT, CATRN  =    1.965    TOTAL FLUX AT TRANSECT=      19.92
MEAN DEPTH= 0.447      MEAN VELOCITY= 0.620    SHAPE-VELOCITY FACTOR= 2.529
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TRANSECT E                 3429.0 METERS FROM OUTFALL
     PARAMETER 2: CHLORIDE
QRIVER=    10.139     BACKGROUND CONC.=  13.000
QEFL   =    0.341     EFFLUENT CONCN.=  358.000
UPSTREAM FLUX=    127.37   EFFLUENT FLUX=    121.95   TOTAL FLUX =     249.33
     VELOCITIES SIMULATED FROM RESISTANCE EQN.
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Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	30.00	0.0	0.0	0.0	0.0	0.0	2.494	1.638
1.83	0.12	0.23	29.00	0.11	0.01	0.38	0.050	0.001	2.411	1.584
3.05	0.24	0.37	29.00	0.33	0.08	2.30	0.083	0.008	2.411	1.584
6.10	0.39	0.51	29.00	1.30	0.50	14.57	0.167	0.050	2.411	1.584
9.14	0.48	0.58	29.00	2.63	1.23	35.69	0.250	0.121	2.411	1.584
10.36	0.57	0.65	28.00	3.27	1.62	46.93	0.283	0.160	2.328	1.529
12.19	0.65	0.71	26.00	4.38	2.38	67.40	0.333	0.235	2.162	1.420
15.24	0.79	0.81	23.00	6.57	4.05	108.21	0.417	0.399	1.912	1.256
18.29	0.79	0.81	19.00	8.97	5.99	148.99	0.500	0.591	1.580	1.038
21.34	0.80	0.82	12.00	11.39	7.96	179.57	0.583	0.786	0.998	0.655
24.38	0.08	0.18	5.00	12.74	8.64	185.29	0.667	0.852	0.416	0.273
28.96	0.08	0.17	1.00	13.10	8.70	185.49	0.792	0.858	0.083	0.055
30.48	0.84	0.85	0.0	13.80	9.06	185.67	0.833	0.893	0.0	0.0
36.58	0.0	0.0	0.0	16.36	10.14	185.67	1.000	1.000	0.0	0.0

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AVG. CONC. JUST BELOW OUTFALL, CAVG=  12.029
AVG. CONC. AT THE TRANSECT, CATRN  =  18.313    TOTAL FLUX AT TRANSECT=    313.04
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TRANSECT E                 3429.0 METERS FROM OUTFALL
     PARAMETER 3: CONDUCTIVITY
QRIVER=    10.139     BACKGROUND CONC.= 340.000
QEFL   =    0.341     EFFLUENT CONCN.= 2257.000
UPSTREAM FLUX=  3331.29   EFFLUENT FLUX=    768.85   TOTAL FLUX =    4100.14
     VELOCITIES SIMULATED FROM RESISTANCE EQN.
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Y	Z	VEL	CONC	SUMA	SUMQ	SUMF	Y/B	QY/QT	C/CAVG	C/CATRN
0.0	0.0	0.0	160.00	0.0	0.0	0.0	0.0	0.0	2.110	1.489
1.83	0.12	0.23	160.00	0.11	0.01	2.07	0.050	0.001	2.110	1.489
3.05	0.24	0.37	160.00	0.33	0.08	12.67	0.083	0.008	2.110	1.489
6.10	0.39	0.51	190.00	1.30	0.50	86.71	0.167	0.050	2.505	1.769
9.14	0.48	0.58	180.00	2.63	1.23	221.43	0.250	0.121	2.374	1.676
10.36	0.57	0.65	170.00	3.27	1.62	290.45	0.283	0.160	2.242	1.582
12.19	0.65	0.71	160.00	4.38	2.38	415.51	0.333	0.235	2.110	1.489
15.24	0.79	0.81	150.00	6.57	4.05	673.70	0.417	0.399	1.978	1.396
18.29	0.79	0.81	95.00	8.97	5.99	911.58	0.500	0.591	1.253	0.884
21.34	0.80	0.82	55.00	11.39	7.96	1059.59	0.583	0.786	0.725	0.512
24.38	0.08	0.18	30.00	12.74	8.64	1088.19	0.667	0.852	0.396	0.279
28.96	0.08	0.17	0.0	13.10	8.70	1089.17	0.792	0.858	0.0	0.0
30.48	0.84	0.85	0.0	13.80	9.06	1089.17	0.833	0.893	0.0	0.0
36.58	0.0	0.0	0.0	16.36	10.14	1089.17	1.000	1.000	0.0	0.0

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AVG. CONC. JUST BELOW OUTFALL, CAVG=  75.834
AVG. CONC. AT THE TRANSECT, CATRN  = 107.428    TOTAL FLUX AT TRANSECT=   4420.45
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TRANSECT E — VARIANCE FROM DIFFERENT METHODS:

PARAMETER	VCMAX	VCN	VUF	VCQ	VPQ
1	47.85	274.52	253.49	20.06	6.05
2	58.77	158.48	213.27	18.76	6.10
3	50.14	149.43	202.97	17.13	5.23

NONDIMENSIONAL VARIANCE   X/B=   93.75   X/H=   7667.0

PARAMETER	VCN/BB	VCN/HH	VUF/BB	VUF/HH	VCQ/QQ
1	0.2052	1372.44	0.1895	1267.27	0.1952
2	0.1185	792.31	0.1594	1066.19	0.1825
3	0.1117	747.08	0.1517	1014.72	0.1667